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Performance assessment of fuzzy logic control systems via stability and robustness measures

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PERFORMANCE ASSESSMENT OF FUZZY LOGIC CONTROL SYSTEMS VIA STABILITY AND ROBUSTNESS MEASURES

A DISSERTATION

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Shehu Sa'id FarinWata

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PERFORMANCE ASSESSMENT OF FUZZY LOGIC CONTROL SYSTEMS VIA STABILITY AND ROBUSTNESS MEASURES

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Date approved by Chairman 8/18/93

DEDICATION

This Dissertation is specifically dedicated to the memory of my beloved paternal grandmother, Hajiya Dangana Rukaiya "Maituwo" FarinWata, who single-handedly took on the role of a mother and father. She was my mentor, my pillar, my educator and my best friend. Her family spirit and generosity knew no boundary. And to her mother, Hajiya Fatima "Maikosai" and sister, Hauwa "Nna Mai Zambu" who brought me up from above the "two beds" in our room, and instilled in me, the best values and sense of discipline, that one could ever hope for, now I realize. And to my maternal grandmother, Hauwa "Aho Maisaka", who was the bravest of them all. I will remain a forever-fortunate recipient of her sense of dignity, care and resourcefulness, generosity, humane heart, and above all, love in its purest form. It was truly a blessing to have been brought up by these beautiful people, who could not be around to see the result of this endeavor. Finally, to my grandfather, Dubagari FarinWata, whom I never got to know, but who gave me the name "Shehu" to mean a person of learning, a teacher; what a name to live up to. I hope I've succeeded in doing that ... Way to go grandpa! and thanks. My love, praises, gratitude, and prayers for eternal peace will continue to be with them, wherever they may be ...

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Shehu Sa'id FarinWata
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PROLOGUE

What Analysis?

Recently, prominent researchers in the area of fuzzy logic control have argued vehemently if a rigorous analysis comparable to the type found in modern control theory is sufficient or even necessary in fuzzy control engineering. Others have pondered, from an industrial point of view, what analyses are really needed in fuzzy logic control. Perhaps, rather surprisingly, stability was not even considered a top priority. Experimentation and testing presided over a rigorous mathematical analysis of stability. The lack of such analyses in fuzzy control has been suggested as the point of attack by many modern control theorists. Indeed, it was proclaimed that, from an industrial point of view, a rigorous analysis of stability may be neither necessary nor sufficient. It suffices to say whether or not a rigorous mathematical analysis of the stability of a given process is carried out in control theory or fuzzy control, the real process needs to be simulated, experimented upon and tested rigorously. Needless to say, no real, physical process works on analytical rigor alone. From this view point, it would seem, indeed, that a rigorous mathematical analysis of stability of the real process is certainly not sufficient, and depending on the process, it may not even be necessary. However, there seems to be a catch here. If such a rigorous analysis is conducted strictly for the sake of what has been termed as "intellectual arrogance" [60], then it may not be useful or even usable, and therefore has a questionable necessity. A useful and usable analysis may be, for example, one that has been employed as a major design tool, for the sake of selecting certain viable system parameters for actual implementation. Such an analysis may be considered necessary but not sufficient. In either case, to uphold a good engineering practice, validation through experimentation and rigorous testing can not be by-passed. The yearning for rigor and analyticity in the proof of stability, and the insistence that it be necessary, in especially fuzzy logic control, has been proclaimed the "***cult of analyticity***" [60].

SUMMARY

This dissertation is concerned with the performance of dynamical systems that are controlled via fuzzy logic and fuzzy sets. Such systems are commonly called fuzzy logic control systems. The theory of fuzzy logic is witnessing an application explosion since the advent of rule-based techniques, by Mamdani and Assilian, which employed Zadeh's fuzzy sets and fuzzy logic. These concepts are introduced as a means of capturing human expertise and also for dealing with uncertainty. The ideas and concepts were quickly applied to ill-defined processes. Until then, a large class of these systems were operated by experienced operators who often achieved excellent results, utilizing information that was, at best, imprecise. Because of the nature of these systems, one sees immediately certain problem areas: 1) It is difficult to ascertain the performance of such systems, especially in an objective manner. 2) The systems' structure makes it difficult to study such performance properties as stability, robustness and controllability in a control theoretic framework. 3) The lack of formalism in the design of these systems has given rise to numerous nonstandard ways of assessing stability, and most of which are dependent on the design approach adopted. 4) Even though in most of the stability analyses of fuzzy systems, robustness of the system is often mentioned and sometimes claimed, a concrete robustness analysis formulated in a systematic framework has been lacking. Controllability in a fuzzy sense is almost not addressed at all. These issues are the motivation for this research.

The objective of the thesis is to develop a viable analytical means of assessing the performance of fuzzy logic control systems. Stability and robustness are considered as measures of the closed loop system's performance. This effort is two-fold. 1). To formulate stability theorems from control theory for analyzing the stability of a fuzzy rule base.

2). To formulate the robustness problem in the framework of modern robust control and develop the condition for robust stability of a wide class of closed-loop fuzzy systems. New stability analysis methods are formulated within the framework of control theory for fuzzy logic control systems. Although the stability analysis developed is an essential contribution, the major contribution of the thesis is the robustness analysis of fuzzy logic control systems. Perhaps, this is why a much larger proportion of the thesis has been devoted to this subject. The thesis addresses the stability of the fuzzy logic control systems from an input-output standpoint. The key to the development of the theory is in the formulation of an input-output mapping; as a stability requirement, the fuzzy logic controller (FLC) should ensure that this mapping is dissipative. In conjunction with stability concerns is the issue of controllability. A controllability condition for fuzzy systems is formulated based on fuzzy rules and fuzzy relations. As a preliminary step for robustness analysis, the concept of fuzzy sensitivity is introduced. This allows two things to be accomplished in the formulated theorem. 1) The essence of fuzziness is captured via a direct utilization of membership functions and therefore fuzzy numbers and fuzzy rules. 2) An approximation was derived for the partial derivative in terms of an equivalent expression employing membership functions of the pertinent variables. This greatly simplifies computation. Finally, a new robustness theorem was then developed in the framework of modern robust control, employing fuzzy sensitivity and a performance index that was formulated as a Lyapunov-like function, which needed to be minimized.

These performance measures were applied to two physical systems for which fuzzy controllers have already been designed. The first is the test-bed fuzzy controller for an automotive engine operating under idle speed conditions. The second is a controller for a missile autopilot's yaw axis. Both performance measures were applied on the test-bed idle speed controller while only the stability analysis was applied to the missile autopilot.

CHAPTER I

INTRODUCTION

1.1 Introduction

The analysis of robustness of fuzzy systems in a control theoretic setting has not been addressed in a concrete fashion by researchers in this area. Although several stability analyses methods are continually being proposed, they tend to have severe limitations. Some of them are only applicable to a linearized model of the plant [9,24,25,28,29]. Others impose certain conditions that may be difficult to satisfy by the general fuzzy systems [9, 26]. A lot more are just not practical. Controllability is almost always taken for granted. Most of the reasons given for the inapplicability of conventional analytic stability and robustness theorems on fuzzy systems were because the design of these controllers mostly employed the operator manual-type if-then rules approach with no mathematical system model to base any analysis upon. This turned out to be a short-sighted view point. E.H. Mamdani, in his plenary speech at the 2nd IEEE International Conference on Fuzzy Systems in San Francisco, March 30, 1993, retracted, apologetically, the original assertion that fuzzy logic control was only suited where a mathematical model of the process was not available. If an approximate mathematical model of the process is available either for design of the fuzzy rules or for analysis, then control theoretic theorems can be formulated to study the behavior of the process, under the applied action of fuzzy control rules. This is the approach taken in this research. Indeed, today, fuzzy control is becoming more and more model-based in an attempt to systematize the design methodology; the original belief is therefore fast dwindling.

1.2 Scope of Thesis

The thesis addresses primarily the issue of stability and robustness of systems that are controlled via fuzzy logic and fuzzy sets. A brief discussion of the historical origin of fuzzy sets and fuzzy logic is given, followed by an overview of fuzzy logic control. This is done to familiarize the general reader with the new and complementary outlook in designing control systems. However, the scope does not extend to the design of fuzzy controllers. In the treatment, it is assumed that a fuzzy controller is available whose performance needs to be evaluated in terms of the measures of stability and robustness. Nevertheless, where necessary, a brief insight is given into the design methodology of the fuzzy controllers that have been employed in the thesis to illustrate the developed theories. Also, because of the tremendous research effort that was put into the development of a systematic fuzzy logic control design methodology, an appendix has been dedicated for this material. The theory developed and measures formulated are general in nature and are applicable to a wide class of plants representable, fairly accurately, by a system of first order differential equations, and which incorporate a fuzzy controller in closed loop. The applicability covers model-based fuzzy controllers as well as fuzzy controllers designed by the operator manual-type if-then rules. In either case, the state space and the control space, or more appropriately, the relevant universes of discourse can be quantified, and an approximate mathematical model is available at least for the purpose of analysis. Even though a fuzzy controller is assumed to be available, a framework for a typical stable rulebase design and analysis is outlined, emphasizing the expected desired topological properties. The first stability theory developed using a Lyapunov-like analysis pertains to systems that are considered, or can be made, dissipative. Thus, discussion is limited to the case where a dissipative mapping can be generated by the fuzzy controls in various regions of the phase plane. The theorem necessitates the induction of such a dissipative mapping by the fuzzy controls as a condition for stability. The second theorem applies to a class of process set-point control systems. From the point of view of their typical operation, the control of these systems is considered to be very fine near the desired set-point and elsewhere very coarse. Based on this significant practical consideration, the theorem classifies the system as being globally dissipative by requiring three things: 1) the set-point be asymptotically stable, 2) the system be dissipative in the neighborhood of the origin and 3) the growth of any nonlinearity sufficiently away from the origin is of order zero. The proof of dissipativeness is shown by employing the Kalman-Yakubovich Lema. Also, though not a major heading in the thesis, the issue of controllability of fuzzy systems has been researched at some length because of its close ties to stability. Controllability conditions are derived for a class of fuzzy control systems that are designed using fuzzy rules and fuzzy relations. Such designs are typically not based on a mathematical model of the process to be controlled. On the robustness front, discussion is limited to the case of small parameter variation while still operating under the same rulebase. External disturbances are considered to be of known origin and are bounded. Control inputs of the 'wildly' varying type are not considered, as a matter of physical reality. The reason being that the rulebase has been designed for the appropriate universe of discourse of the system's inputs which are known and bounded. Moreover, the outputs have been proved to be bounded in their respective universes of discourse. That leaves 1) parameter variation due to aging, wear, drift, etc., which could give rise to a reasonable variation of a closely related family of plants, all other things being equal. 2) external disturbance which is known and bounded.

Discussion on robustness is limited to robust stability. However, the theorem is formulated to incorporate the global desired performance objective, which is the system's convergence to a design-specified set-point. This is done by minimizing an objective function that is formulated a Lyapunov-like function. Even though such performance quantities as speed of response, settling time, overshoot have not been explicitly specified, it is believed that the system's stable operation in the presence of model and parameter variations, and an external disturbance, is indicative of some robust performance, as quantified in the theory. Singular values are used as the mechanism for the quantification.

1.3 Generalities and Applications: A Philosophy

This section has been added in order to clarify certain important facts. As already stated, the theories developed in the thesis are very general. Fuzzy controllers are designed in a very unconventional way. For the most part, they are system-specific entailing a very tasking procedure. Recently, however, the design methodology is seeing a lot of systemization. This allows plants with similar mathematical model structure, as alluded to in the scope of the thesis, to be interchanged with only a slight modification. The modification usually involves redefining the universes of discourse for the relevant states and controls, and occasionally, adjusting the support of the membership functions. Overall, the structure of the controller remains the same. That is the essence of a systematic design methodology. For instance, on the premise part of a fuzzy controller, the states can be interchanged with the actual outputs, if these are different, and the entire if-then structure, method of fuzzification, inferencing and actual control remain intact. One may not have this liberty with modern controller design.

Take for instance where a state feedback is assumed. There is a host of techniques for eigenstructure assignment. Now try replacing the states with the actual outputs, if these are different, and you have an output feedback problem which, in most cases, may not be treated purely with the gadgets of a state feedback problem. The point here is that fuzzy controllers are not the every day garden variety controllers expressible in a neat closed form that is very general. What suffices, for the purpose of application or illustration, is a good, typical fuzzy controller that has been designed in the systematic way alluded to. This is then used as a test-bed for evaluating the various performance theories. This is the step taken in this research. Once such a solid fuzzy controller has been realized for a specific system, as is the case in this research, using a systematic design methodology, the design variation from one system to another in not significant. For instance the same design tool may be used to control say, a missile autopilot, an inverted pendulum, a simple servomotor, the dynamic interaction of demand and supply in a competitive market situation, or even the dynamics of two species of ants in a closed environment trying to annihilate each other. Clearly, the list can go on ad infinitum, and will still not render the application domain general. Thus, using any number of mathematical models from the infinite collection of models only shows how general the fuzzy controller is, and may not constitute a generalization of any performance theory, since the controller, as discussed above, is structurally invariant under a systematic design methodology. The performance theories, on the other hand should be developed in the most general way possible, by defining a broad class of plants for which the same fuzzy controller is functional. This way, one ends up with a general theory for a very general fuzzy controller. This is the attitude taken in this thesis; it is therefore only a matter of flavor, rather than of generality, that more than one plant model is used in the application examples.

1.4 Outline of Thesis

The thesis is organized as follows:

Chapter I states the motivation and objective of the research and introduces the contributions in this area. The scope of the thesis are discussed here followed by comments about generalities and applications.

Chapter II gives a brief introduction to fuzzy sets and fuzzy logic followed by an introduction to fuzzy logic control. Current work in the area of stability and robustness are reviewed in this chapter.

Chapter III discusses some topological properties of a stable rule base. Some insights are given into the design of a test-bed fuzzy controller that has been widely used in the examples in the thesis. Here also, the issue of controllability of fuzzy control systems is discussed. In particular controllability conditions for fuzzy relational systems are derived.

Chapter IV compactifies the representation methods for fuzzy "control laws" and establishes some formal terminology. The first stability theory pertaining to dissipative systems is discussed in this chapter followed by an application on the test-bed fuzzy controller.

Chapter V. Here, the second stability theory is developed for the nearly-linear process set-point control systems. A simple proof of dissipitativeness is illustrated on a linearized model of the test bed controller using the Kalman-Yakubovich Lema.

Chapter VI introduces the concept of input-output sensitivity and robustness. The framework for fuzzy robustness analysis is established followed by a statement of the robustness problem. The underlying hypothesis are discussed and the robustness theory is developed for a decoupled system. Later on in the chapter, an estimate of the measure of the coupling between states is obtained, and the theory is generalized accordingly. From the conditions for robust stability, inequalities are derived in terms of membership functions of the outputs and the parameters. Bounds on the sensitivities are also established. Finally, the robustness measure is formulated by an example on the test-bed fuzzy controller for the automotive engine.

Chapter VII summarizes the contributions of the thesis and make recommendations on future research issues.

Five appendices are added:

Appendix I contains detailed material on fuzzy logic fundamentals. A lot of definitions and examples are provided on fuzzy logic and fuzzy sets.

Appendix II gives a brief and general overview of the fuzzy logic control process. The pertinent terminology and procedures are explained.

Appendix III provides detailed material on fuzzy feedback systems representation. This is a fuzzy dynamical system description that has been widely used to develop early non model-based stability theorems and identification schemes.

Appendix IV describes a systematic design methodology that was developed here at Georgia Tech. This has been a major research effort in the development of the assessment tools developed in this thesis. The design methodology was applied to the test-bed fuzzy controller for an automotive engine's idle speed control, and this test-bed controller is used throughout the thesis to illustrate the developed theories.

Appendix V provides a "crash course" on elementary Lyapunov's stability theory. A biography section is also added followed by a personal vita page.

CHAPTER II

BACKGROUND

2.1 Introduction

In Fuzzy theory, all things are considered as a matter of degree. It reduces the black-white logic and mathematics to special limiting cases of gray relationships. Many of the laws of traditional or binary logic are violated. In particular, the law of excluded middle *either A or not-A* is drastically relaxed in the sense that several possibilities are now allowed in between. Mathematically, fuzziness means multivaluedness or multivalence. [Rosser 1952, Rescher 1969]. It stemmed from the Heisenberg position-momentum uncertainty principle in quantum mechanics [Birkhoff 1963]. The multivalued fuzziness is an indication of degrees of indeterminacy or vagueness, ambiguity, partial occurrence of events or relations.

In 1965, system scientist Lotfi Zadeh published the paper "Fuzzy Sets" [1], that formally developed multivalued set theory and introduced the term 'Fuzzy' into the literature. This gave birth to a new wave of interest in multivalued mathematical structures from Fuzzy systems to Fuzzy topologies. We are now into yet another wave, that of the application of fuzzy logic in the design of commercial products. Zadeh extended the bivalent indicator function I_A of nonfuzzy (crisp) subset A of X, $I_A(x) = 1$ if $x \in A$ or 0 if $x \notin A$, to a multivalued indicator or membership function, $\mu_A : X \rightarrow [0,1]$. A concise treatment of fundamental concepts in fuzzy set and fuzzy logic can be found in [1,2] and Appendix III.

2.1.1 Fuzzy Systems

A mutivalued or fuzzy set can be considered as points in the unit n-dimensional cube, I^n. where $I = [0,1]$. Within the cube one is interested in the distance between points which led to measures of the size and fuzziness of a fuzzy set and the concept of fuzzy subsethood. A fuzzy set defines a point in a cube while a fuzzy system defines a mapping between cubes. This between-cube theory is fuzzy system theory. A fuzzy system S maps fuzzy sets I^n to fuzzy sets I^p. The n-dimensional unit hypercube I^n contains all the fuzzy subsets of the domain space or the universe of discourse of the input, $X = \{x_1,\ldots,x_n\}$ while the I^p contains all the fuzzy subsets of the range space or the output universe of discourse, $Y = \{y_1,\ldots,y_p\}$. These continuous fuzzy systems map close inputs to close outputs. Some authors referred to them as fuzzy associative memories, or FAMs [65].

2.2 Fuzzy Logic Controllers

These are numerical systems enriched with expert knowledge as a natural language. In general, they consist of a bank of rules or partial implications or approximate heuristics, operating in parallel, and executing to different degrees. Each rule is a set-level implication of the form (IF A_i-THEN B_i). It represents ambiguous expert's knowledge or learned input-output transformation. Fuzzy logic control is based on fuzzy logic. Fuzzy logic itself is much closer in spirit to human thinking and natural language than the traditional binary logic. It provides an effective means of capturing the approximate, inexact nature of the real world. In this regard, the essential elements of the fuzzy logic controller (FLC) are a set of linguistic control rules of the type given above. The rules are related by the dual concepts of fuzzy implication and the compositional rule of inference [[4], and Appendix I].

Therefore, the FLC essentially provides an algorithm which will transform the linguistic control strategy based on expert knowledge into an automatic control strategy. The fuzzy logic control methodology appears very useful when the processes are too complex for analysis by conventional qualitative techniques or when the available sources of information are expressed in qualitative linguistic terms and naturally incorporates some elements of uncertainty. In other words, the conventional quantitative techniques of system analysis are intrinsically unsuited for dealing with systems whose complexity can be compared to that of humanistic systems [2]. This contention was made evident by what Zadeh called the *principle of incompatibility* which is stated informally below [2]:

> As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics.
>
> *Corollary*: The closer one looks at a real-world problem, the fuzzier becomes its solution.

While almost all designers of fuzzy logic control systems agree that the theoretical foundation of these systems was laid down by Zadeh in his paper [2], the applied research was originated by E. Mamdani and his students, in particular, the work by Mamdani and Assilian [1975]. The following is an early statement from [Kickert and Mamdani, 1978]:

> The basic idea behind this approach was to incorporate the "experience" of a human process operator in the design of the controller. From a set of linguistic rules which describe the operator's control strategy, a control algorithm is constructed where the words are defined as fuzzy sets. The main advantages of this approach seem to be the possibility of implementing "rule of thumb" experience, intuition, heuristics, and the fact that it does not need a model of the process. [Kickert and Mamdani, 1978]

The last sentence regarding the need for a model of the process has already been recanted by Mamdani [60], as more and more model-based fuzzy logic controllers are being proposed in an effort to systematize the design methodology.

2.2.1 Fuzzy Logic Control Design

As alluded to under "scope of thesis", the thesis does not specifically address the design of fuzzy logic controllers. For the development of the fuzzy logic control system assessment tools, it was assumed that a fuzzy controller is available for a given process. However, because the analysis and design of these systems are intimately related, a lot of insight into the design process is given where possible, throughout the thesis. In particular, Chapter III gives an insight into a stable rulebase design from a general perspective. Moreover, in this research, a substantial amount of time was spent in systematizing the design methodology of fuzzy logic control systems, the result of which is the test-bed fuzzy control system that has been widely used in the thesis. Because of this, a whole section pertaining to this systematic design is provided in Appendix II. Also, a general description of the fuzzy logic control process is placed in Appendix I.

At this point a distinction will be made among the two prevalent design methodologies:

The first is the operator manual-type design where the control rules are taken out of the operating manual of the process. This is also regarded as the *Heuristic approach* where the rules have been put together by expert operators who have become so well familiar with the process dynamic behavior, and experienced in controlling it. This is the earliest form of fuzzy control that was characteristic of the pioneering work of Mamdani and Assilian on the steam engine [1978], Ostergaard, King, Umbers and Lembesis on the cement kiln operation [1977, 1988], [Sugeno 1985, 1989] and several others. The second approach is systematic and all-encompassing in that an approximate model of the process to be controlled is employed in the design in addition to heuristics. The work done here at Georgia Tech is one such approach [61].

2.3 Performance of Fuzzy Logic Control Systems

The purpose in a controller design is ultimately to use the controller in a closed-loop fashion in conjunction with the model of the process to be controlled. The objective is to meet a certain design specification, be it tracking a prescribed trajectory or regulating the system's behavior according to a given set-point. Some of the ways to ensure a proper system behavior is by monitoring certain observable quantities through a variety of engineering instrumentation devices. In all of this effort, a stable operation of the closed-loop system is perhaps the most desirable property. However, stability alone is not a sufficient measure of the system's performance. The fact that one does not observe things blowing up does not mean that the specific design objectives are being met.

Stability, for instance, does not give on a first hand basis, information about rise time, overshoot and the like [75]. A good performance measure should give bounds on the relevant design parameters in term of inequalities [75]. Stability does not also tell the designer what size and type of perturbation the system can accommodate or if it can accomodate any perturbations at all, be it structural, parametric or noise. However, the sizes of a lot of these quantities can be inferred when one starts talking about "stability margin". This immediately calls into play another important measure of the system's performance, which is robustness. With this tool, values of specific design parameters can be perturbed about their nominal values and within their possible and physical variational limits. Qualitative and quantitative statements may be made as to the system's ability to "cope" with these perturbations. Noise can be injected, a whole family of closely related process models can be used interchangeably, with the same controller in the loop, and the system's performance can now be studied in an elaborate manner. It needs to be mention, nonetheless, that no matter what analysis has been resorted to, the "modest attitude" to engineering must be taken: rigorous testing and experimentation. Thus, from an engineering point of view, an adequate assessment of a system's performance study should consist of 1) stability, 2) robustness and 3) validation through rigorous testing. Even from the analytical nature of 1 and 2 certain lessons can be learned. For instance, where the system fails to be robust, the non robust parameter space can be tracked down and the pertinent parameters identified and redesigned for better performance. What this suggests is that, though these take the form of an analysis tool, they should be comprehensible, usable and useful, as long as the objective is to build a physical system whose performance is to be analyzed. This is the emphasis stressed on the performance analysis of fuzzy logic control systems undertaken in this research.

The next section, discusses how the above issues are currently being addressed.

2.3.1 Current Approaches to Stability Analysis

There are indeed several techniques for analyzing the stability of fuzzy control systems that can be found in the literature. The problem is the lack of a standard, all-encompassing technique that can be used for both fuzzy controllers designed using the operator if-then rules alone and those designed based on an approximate mathematical model of the process. In either case, the system's relational matrix can be derived from the fuzzy control rules, and so the stability analyses should be "topologically" equivalent. The lack of a standard systematic design methodology for fuzzy controllers also compounds the problem. The list below gives some of the more acceptable techniques and their associated shortcomings.

1. Multi-level relay approach:

Kickert and Mamdani developed a multi-level relay model of fuzzy control and further used classical Describing Function(DF) techniques associated with these types on nonlinear elements to study the stability problem.

Drawbacks:

The DF method is generally used to determine the existence of periodic oscillations in nonlinear systems; it is an approximate method at best and does not directly address the stability issue.

2. Systematic Mapping:

Brae/Rutherford developed the notion of granularized state space and proposed a method of analysis which is based on systematic mapping of the process state, using control rules as transition functions.

Drawbacks:

It appears that this approach only confirms what was a priori assumed at the design stage.

3. Energetistic Stability:

Kiszka et al. proposed this notion which takes into account such characteristics as membership function shape, spread etc. Robustness and stability are discussed within the same framework.

Drawback:

Such notions of stability and robustness are intertwined, and as a result, are difficult to address independently.

4. Cell-to-cell Mapping:

Chen applied Hsu's cell-to-cell mapping to the stability analysis of FCS.

Drawback:

This approach requires an accurate mathematical model of the process and further, it is only locally accurate.

5. Circle Criterion on Sector Bounds:

Ray et al used the circle criterion approach on the resulting output of the FCS which has been approximated as a multi-level relay. A sector bound is placed on this output and stability of the inverse system is discussed using Rosenbrock's diagonal dominance and inverse Nyquist array. This approach also addresses stability and robustness within the same framework.

Drawback:

Multilevel relay approximation may not be possible; it assumes that membership functions are symmetrical about their maximum value or about the origin, and that the max-value defuzzification approach is employed.

Also, on dealing with the inverse system, care must be exercised not to introduce non-minimum phase zeros. A strong effect of high frequency coupling may weaken the diagonal dominance (decoupling) assumption.

6. Direct Lyapunov Approach: Chen

Others have used directly the Lyapunov approach on the system and derived the stable controller expression which was later fuzzified.

Drawback: An accurate mathematical model is needed. One must also be careful what to fuzzify. It is more of a design approach than a stability analysis. The closed-loop system still needs to be analyzed for stability under the fuzzified control.

7. Lyapunov-in-a- cell: S. Chand

This method uses the Lyapunov stability criterion. The state space is partitioned into cells so as to compute the minimum and maximum bounds on the controller output in the individual cells. Based on these bounds, an attempt is made to verify that the Lyapunov function is monotonically decreasing in every cell.

Drawback: One minor drawback, depending on the system, is that the system is assumed linear in each cell. Also, since there are two control actions for every cell, a difficulty arises as to what point or points should be used for transitioning purposes, and whether the minimum or the maximum control should be used to this effect. Another constraint imposed by this method is the completeness of the rules. That is, a control action needs be inferred for every point in the state space. The method is effective for regions in the state space distant from the origin, and requires increasingly finer partitions for regions near the origin. Other efforts in this area include the works of Sugeno [24], Aracil [18], Pedrycz [7], Kania [27, Maeda [47], etc.

2.3.2 Current Approaches to Robustness Analyses

Ad hoc Approach

This procedure is particularly suited where a mathematical model of the process is not available, [Kosko, 1991]. It entails the following.

- Rule Modification: deletion of rules, introduction of sabotage rules.
- Modification of membership functions
  - modification of support, spread, height, type etc. of membership functions
  - increase/decrease the number of linguistic terms
- Introduction of Disturbance

Conclusion for Robustness

- The fuzzy controller is considered robust if it remains stable and meets the performance objective under a combination of the above scenarios.

Drawback: This approach, though somewhat practical, can be very subjective. It also lacks a systematic base.

Systematic Approach

On the theoretical side, there are a handful of approaches that have been proposed. In general, just about every paper in fuzzy controller design or stability analysis has something to say about the robustness of the fuzzy controller viz a vi the conservative nature of the max-min operation and the choice of say, a Gaussian-type membership function [22]. Others have given rigorous treatment on the robustness of fuzzy operators and not in a control system setting [21]. The one method that used some control theoretic concept is the work of Ray and

Majumder [9]. The circle criterion is used to obtain some stability margin of a linear plant that is driven by a fuzzy controller. The limitations of this approach has already been pointed out.

CHAPTER III

A FRAMEWORK FOR STABLE RULEBASE DESIGN AND ANALYSIS

3.1 Introduction

Given a fuzzy control rulebase, an analytical framework for verifying the stability claim is discussed. This analysis is approached from a topological point of view and is considered necessary before other formal theorems may be applied. Thus, in the development, it is assumed that the fuzzy controller has been designed in a more or less systematic way. Most of such systematic design procedures involve the partitioning of the relevant state space into suitable regions or cells, each with its fuzzy control rule. The origin will be considered to be in the target or goal cell. The objective is therefore to converge asymptotically to this unique origin from any point in the phase plane. The insight given here is applicable to the variety of systematic design methods for fuzzy logic control systems. However, the manner in which the desired target point is singled out from a host of quasi-target points depends on the design criteria. In [61] for example, this is done algorithmically through a heuristic search of a large amount of cell-to-cell transition data. The selection criteria could be minimum time, minimum squared error or minimum effort. In this chapter, a brief description of the rulebase design is given, followed by a topological analysis, in order to validate the stability claim that has been assumed a priori.

A theory for controllability is also introduced for the general fuzzy system, in particular, for fuzzy systems that are designed strictly from fuzzy rules and fuzzy relations.

3.2 Insight for a Stable Rulebase Design

Suppose an approximate mathematical model of the plant to be controlled is available, and is representable as a member of the class:

$$\dot{x} = f(x(t), u(t), d), \; x \in \mathbb{R}^n, u \in \mathbb{R}^m, d \in \mathbb{R}, t \in \mathbb{R}_+ \tag{3.1}$$

where $x_1, \ldots, x_n$ are the states. These are assumed to be available for measurement either directly or indirectly, otherwise they can be estimated. $f_1, \ldots, f_n$ are generally nonlinear mappings; and $u_1, \ldots, u_m$ are the control inputs. d is an additive scalar disturbance. Furthermore, let the admissible controls and states be given as:

$$u \in [u_{\min}, u_{\max}], \;\; x \in [x_{\min}, x_{\max}], \;\; d \in [d_{\min}, d_{dmax}] \tag{3.2}$$

These form the supports of the respective membership functions for the controls, the states and the disturbance. If the desired state is elected to be the nominal, no-load, equilibrium point, x_0, then the trajectory error can be expressed as:

$$e \triangleq x_0 - x, \; \in \mathbb{R}^n \tag{3.3}$$

The nominal (equilibrium) operating point is typically determined on the basis of operating conditions, minimum energy dissipation, etc.

Since the intent is to display the fuzzy control rules on a phase plane, it is more convenient to consider $x \in \mathbb{R}^2$. Let also $u \in \mathbb{R}^2$. For more than 2 state variables, several phase planes can still be constructed by taking two variables at a time. A hypercubic structure is usually employed [50].

3.2.1 The Fuzzy Controller

Depending on the observed behavior of the system via prior simulation and some expert knowledge of the particular system, membership functions are designed for the states and controls. This is also where heuristics are used in selecting the type of membership function, be it triangular, bell-shaped or trapezoidal. The number of linguistic labels to be used such as small, medium, large, etc., are also determined at this stage. Membership functions can be of the continuous or discrete type. [Appendix II]. Usually, to reduce the computational task, the admissible control components are discretized into manageable levels. A detailed discussion of a fuzzy logic controller design is outside the scope of the thesis. However, an in-depth description of a particular systematic design methodology will be found in [61].

From the quantized, admissible controls for u_1 and u_2, finite representative constant values are selected and denoted as $u_{1i}(k)$ and $u_{2j}(k)$. These crisp numbers are fuzzified unless the performance of the controller with crisp inputs is satisfactory. Moreover, finite representative points in the state space are chosen to anticipate the trajectories from one subspace to another. The center-points of each cell are often used for computational simplicity. More on the center-point method can be found in [31]. The countless trajectories are called a 'manifold' and one set of data is collected by setting u_{1i} and u_{2j} constant. Such data are repeatedly collected as they are used next in obtaining a rulebase for feedback regulation of x_1 and x_2.

The m-th and n-th intervals of x_1 and x_2 are denoted by E_{1m} and E_{2n} respectively. A cell is defined linguistically as:

$$L_{mn} = E_{1m} \, X \, E_{2n}$$

and then L_{mn} is a finite region in the state space. By applying the fixed controls u_{1i} and u_{2j}, a set of transitional relations are obtained such as, for example:

$$(u_{11}, u_{21}): L_{12} \rightarrow L_{34}, \quad (u_{11}, u_{22}): L_{34} \rightarrow L_{52}, \ldots, (u_{13}, u_{23}) : L_{25} \rightarrow L_{73}$$

(u_{1i}, u_{2j}) is continually changed and a finite number of transitional relations are gathered exhaustively. When the information above is stored, such performance quantities as the required transition time, expended energy and squared error are determined and attached for each transition, together with the control input-pair that is applied while in L_{mn} in order to transition to the next cell-group. On changing the pair, (u_{1i},u_{2j}), one might end up in the same L_{mn} as the source and also for the next transition. This is called an 'invariant manifold'. To avoid an infinite transit time while in such a cell-group, a time limit was set after which system evolution in the cell-group space resumes. It is therefore clear that the target cell-group L_{mn} (the specified goal) must be an invariant manifold for some (u_{1i}, u_{2j})-pair. This is also necessary for convergence and asymptotic stability. This is equivalent to the 'reachability condition' in modern control theory. The target is denoted by L^*. The vector fields of the nonlinear dynamic system are utilized and use is made again of fuzzy optimality concepts to obtain the optimal fuzzy controls for each cell-group. The general form of the rules is given below:

If (e_1 is F_1) & (e_2 is F_2) & … & (e_n is F_n) THEN
(u_1 is G_1) & (u_2 is G_2) & … & (u_m is G_m)

where F_i and G_i are the consequent and premise linguistic terms respectively. Henceforth, the fuzzy logic controller is referred to as the *final rulebase*, the set of If-Then rules that has been designed for a given plant with the desired objective in mind. Figure 3.1 shows a block diagram of the FLC system structure.

Scalar Disturbance Compensation

The external disturbance d is handled in a variety of ways. One way is to include it in the premise part of the rule but that increases the computational complexity. A more efficient way is to compensate for this by an upper level controller. When an external disturbance comes on, the dynamic equations are resolved for a new equilibrium point. Inferencing proceeds then based on this new equilibrium point. The control action necessary to equilibrate at this new point is adjusted accordingly. The upper level controller may take the following form:

If $d \in (d_{min}, d_{max})$ then $u = u_{FLC} \pm$ constant value.

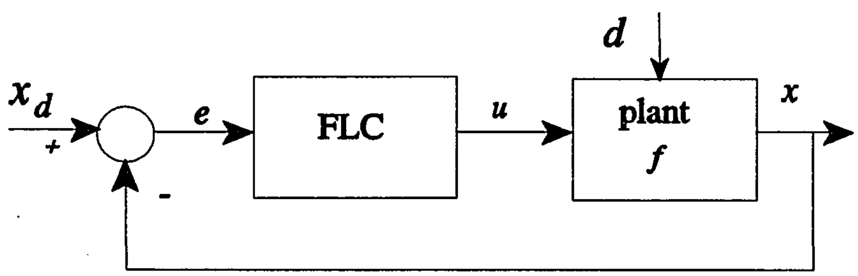


Figure 3.1 Fuzzy Logic Control System Structure

3.3 Analysis

Proposition 3.1: Let p $\in$ P be the nominal, non-parametrized non-linear plant given by:

$$p : \dot{x} = f(x,u,d),\ x \in \mathbb{R}^n,\ u \in \mathbb{R}^m,\ and\ t \in \mathbb{R}^+\ and \tag{3.4}$$

f: $\mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^+ \rightarrow \mathbb{R}^n$, a real mapping.

Let the desired state, x_d be reachable $\forall\ u_i \in U \subset F(U)$, and $\forall\ x_0 \in X \subset F(X)$, where:

F(U): set of feasible inputs, F(X): set of feasible states, $F^*(\mathbb{R})$: set of fuzzy numbers.

<u>Claim</u>: The fuzzy controller k = FLC designed for the plant p in (3.4) under the input and state quantizations Qtz(U) and Qtz(X), respectively, is stable, $\forall\ x_0 \in F(X)$.

<u>*Method of Proof*</u>:

Consider the state space as being $F(X) \subset \mathbb{R}^2$ with quantization Qtz(X): $E_{1n} \times E_{2m}$

$x_1 \in E_{1n}$ and $x_2 \in E_{2m}$. The state space is partitioned into cells L_{mn} according to:

$$L_{mn}(k)\ :\ \begin{cases} x_{1i\min} \leq x_{1i} \leq x_{1i\max} \\ x_{2j\min} \leq x_{2j} \leq x_{2j\max} \end{cases} \tag{3.5}$$

The point that gets mapped to $L_{mn}(k+1)$ is given by:

$$x_{mnc} = \left(\frac{x_{1i\min} + x_{1i\max}}{2},\ \frac{x_{2j\min} + x_{2j\max}}{2}\right) \tag{3.6}$$

which is the center point of cell $L_{mn}(k)$.

Define t_{mn} as the transition time in going from cell $L_{mn}(k)$ to cell $L_{mn}(k+1)$, and t_{max} as the maximum transit time while in cell $L_{mn}(k)$, $\forall$ m,n. Associated with each point is a control pair generated by the search procedure. In particular, associated with the center point x_{mnc}, is the control pair, $\mathbf{u}_{mn} = (u_{1i}, u_{2j})$ such that the following rule holds:

If $(x_{mnc}(k), \mathbf{u}_{mn}(k))$ then $x_{mnc}(k+1)$

This may be expressed as the fuzzy mapping:

$$f(x_{mn}(k)) = L_{mn}(k+1) = x_{mn}(k+1) \tag{3.7}$$

Suppose the current transit time is $t_k > t_{max}$. Then cell $L_{mn}(k)$ is considered to be a sink cell or a local minimum. A new set of feasible inputs is then defined as:

$$F(U) \setminus \{u_{mn}\}, \text{ where } \{u_{mn}\} = \{ (u_1,u_2) \ni f(x_{mn}(k) = x_{mn}(k+1) \Rightarrow t_k > t_{max}\}.$$

This is continued until $F(U) \setminus (u_{1i}, u_{2j}) \triangleq \tilde{F}(u)$. Similarly, if for some $h > 0$, and $t_k < t_{max}$, $f(x_{mn}(k+h)) = x_{mn}(k)$ ($\neq x_d$), then a periodic motion of period h results. Suppose for some $(u_{1i},u_{2j}) \in \tilde{F}(u)$, and $\forall\ x_{mn}(0) \Rightarrow x_{mn}(k+1) \in \{ X \setminus F(X)\} \neq \varnothing$, which is the set of non-feasible states or divergent cells; then the points $x_{mn}(k)$, $k > 0$, are divergent points and they, therefore, give rise to divergent cells, $L_{mn}(k)$, $k > 0$.

Presumably, sink cells, periodic motions and divergent cells have been removed algorithmically before the final rulebase is established. The remaining space, F(X), is therefore a compact invariant, with its associated chosen control set, $\tilde{F}(u)$. By definition, this final rulebase is the fuzzy controller.

The fuzzy mapping in (3.7) is cell-wise continuous in the compact region (3.5). It is therefore bounded. So, what results is a union of bounded mappings in F(X).

Theorem 3.1: If f(x) is fuzzy continuous on $[a, b]^*$ then f(x) is bounded on $[a, b]^*$ [52].

We can therefore form a connected, smooth mapping, given an arbitrary initial point in F(X). This implies, as a consequence of a theorem, that one can form a sequence of these continuous functions which converge uniformly to *f*. Naturally, one would expect that a continuous function can be uniformly approximated by simple functions which are also continuous. This is indeed a consequence of the Stone-Weirstrass theorem. In the rulebase or the fuzzy logic controller design, one is interested in convergence to the value of $f(x_{mn}^*)$, where x_{mn}^* is the target point $\in L^*$. Not only that, it is desired that this value be the fixed point or invariant manifold, ie:

$$f(x_{mn}^k) = x_{mn}^k = x_{mn}, \ \forall k. \tag{3.8}$$

It turns out that, by design, the origin which has been defined to be in the target cell is such a point, ie., $x_{mn} = x_d$. The existence and uniqueness of a solution to (3.4) is ensured by the continuity of the function *f* and its derivative *df/dx*. Furthermore, the above fixed point exists, for it is determined according to the set below:

$$\{\dot{x}=0, (u_1,\dots,u_n) \in F(u),\ x_1=x_{1d},\dots,\ x_n=x_{nd},\ d=0\}. \tag{3.9}$$

For any x, this defines the steady state or static map. Thus along any of the smooth trajectories, the subsequence $\{ f(x_{mn}^k) \}$ can be formed and can be shown to be convergent. The target point defined in (3.8) is therefore reachable. In particular, given p, q > N ∈ **N**, it can be shown that:

$$\|f(x_{mn}{}^p) - f(x_{mn}{}^q)\| \leq c \, \|x_{mn}{}^p - x_{mn}{}^q\| \quad (3.10)$$

is a contraction for c < 1. By a fixed point theorem, f has a unique fixed point. Note that the existence of a fixed point, and not its uniqueness suffices for us, as long as the point is such that:

$$\|x_{mn}(k) - x_{mn}{}^*\| < \varepsilon \,, \forall k. \quad (3.11)$$

where ε is the specified tolerance. The set in (3.11) defines the target region, L^*. Indeed, for all x solutions of (3.4) that fall in the set, there is only one point that is considered representative of the entire cell, which is the center point $x_{mnc} = x_{mn}{}^* = x_d$, in which case, the uniqueness part of the theorem is recovered. Thus, continuity, the existence of a fixed point and convergence to the region defined by (3.11) for $\mathbf{u}_{mn} \in F(U)$, prove the stability convergence of the fuzzy logic controller, ∀ x(0) ∈ F(X) ■

Note that the smaller the cell partition, the higher the precision of avoiding periodic motions. However, the intent is not to make this prohibitively small as to destroy the fuzziness induced by the cell definition (3.5). Also, a major requirement in this exposition, though not stated, is the completion of the rulebase. That is, a control rule must be inferred for every point in the feasible space, F(X). More will be said about rulebase completion in Chapter IV. It is absolutely necessary to show that the rulebase thus designed, with $d = 0$, is stable before any perturbations internal or external are injected. This is equivalent to guaranteeing nominal stability of the fuzzy control system.

3.4 A Controllability Analysis

In the design of fuzzy logic controllers where a mathematical model is not available, an input-output identification scheme is usually employed to formulate the expression of the fuzzy dynamical system [43]. Consequently, one obtains a set of if-then rules that adequately describes the dynamic behavior of the process. These rules are often employed to form the fuzzy relational matrix for the process. Even in the case of the systematic design procedures already outlined, such a relational description of the system can be realized from the resulting rulebase, and the stable cell transitions as can be identified from the phase plane. However, if all one has are the set of rules and the fuzzy relations, it would be reassuring to be able to infer something about the controllability of the fuzzy system. This section develops a theoretical framework for determining the controllability of fuzzy controlled systems. It is targeted at finding a controllability index or condition for fuzzy logic systems, similar to the one for linear time-invariant (LTI) systems or the one formulated for nonlinear system in companion form using Lie Algebra, in modern control theory. Such an index is specified in terms of key fuzzy quantities of the systems, such as, for example, the system's fuzzy relational matrices and the like.

The issue of controllability plays a very important role in the system's control, and in any situation where the objective is to transfer the system from one state to another within a finite time interval, by utilizing a certain control history. The existence of such a control history, that is feasible, becomes a prime concern of the system designer. To begin, let us recapitulate below the familiar results from linear time invariant systems (LTI). The approach for nonlinear systems can be found in [63],[72],[79].

3.4.1 LTI Systems

Consider the discrete-time system expressed by:

$$x(k+1) = \Gamma x(k) + Gu(k), \quad y(k) = Cx(k), \quad x(0) = x_0 \tag{3.12}$$

with $x \in \mathbf{R}^n$, $u \in \mathbf{R}$, and the matrices are of appropriate dimensions. One wishes to know if there exists a control sequence $\{ u_n \}$ such that the system can be driven from x(0) to x(k), within finite [0, k]. By recursively solving (3.12), the solution below is obtained:

$$x(k) = \Gamma^k x(0) + [G \ \Gamma G \ \Gamma^2 G \ldots \Gamma^{k-1} G] \begin{bmatrix} u(k-1) \\ \vdots \\ u(2) \\ u(1) \\ u(0) \end{bmatrix} \tag{3.13}$$

The transfer of states is achieved if the controllability matrix, U_k, has full rank where this is defined as:

$$U_k = [G \quad \Gamma G \quad \Gamma^2 G \quad \dots \quad \Gamma^{k-1} G] \qquad (3.14)$$

This being the case, one can place the poles of the system in (3.12) arbitrarily as desired [Wonham].

3.4.2 Fuzzy Systems

Consider the fuzzy system defined by:

$$\tilde{x}(k+1) = f(\tilde{x}_k, u_k, k) \qquad (3.15)$$

where the variables are fuzzy sets. Let F(X) denote the set of feasible states and F(U) denote the set of feasible control inputs. In other words, the application of any $u \in F(U)$ leads to an invariant set $F(X) \subset X, \forall x \in X$. Note that for practical design consideration, F(X) is per the design objective, and is not the trivial invariant set. Consider the fuzzy dynamical system given in Figure 3.2.

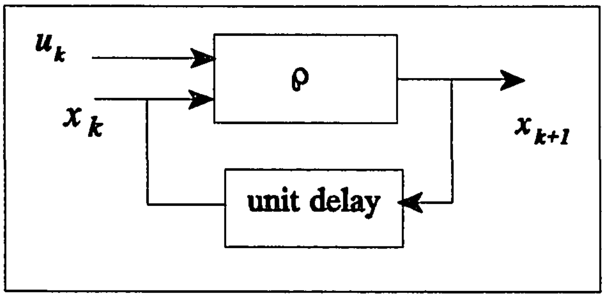


Figure 3.2 Fuzzy Dynamical System

The system may be described by the fuzzy rule below.

Rule i: If $x^i(k) = Lx$ *and* $u^i(k) = Lu$ then $x^i(k+1) = Lx$.

where Lx and Lu are the linguistic fuzzy sets for x and u respectively. In terms of the fuzzy relation, this may be expressed as:

$$\wp = \bigvee_{i=1}^{s} \{u^i(k) \wedge x^i(k) \wedge x^i(k+1)\} = u(k) \circ x(k) \circ x(k+1) \tag{3.16}$$

where $\wedge$, $\vee$ are the min and max operations respectively, and s is the number of rules.

<u>Problem Statement</u>:

The question being asked here may be posed equivalently as: does there always exist a certain fuzzily derived input sequence, $\{ u_n \}$, for $n \in [0, k]$, $k \leq s$, where s is the number of rules, such that given the initial state x_0, the fuzzy controller in the closed loop system can drive the system to $x_k \in F(X)$, for a finite number of rules, with the system's dynamics described by a fuzzy relation of the type (3.15), given that the fuzzy sets have a finite cardinality?

3.4.3 Heuristic Rule Decomposition

The rule above may be decomposed in the following way:

***Rule i*:** [if x(k) = Lx then x(k+1) = Lx] *and*

[if u(k) = Lu] then x(k) = Lx.

The fuzzy relational matrix, $\mathcal{P}$, becomes:

$$\begin{aligned}\mathcal{P} &= [x^d(k+1) \circ x^d(k)] \wedge [u(k) \circ x^d(k)] \\ &= [x^d(k+1)]_{col} \circ [\, x^d(k)\,]_{row} \wedge [u(k)]_{col} \circ [\, x^d(k)\,]_{row}\end{aligned} \tag{3.17}$$

where the composition, $x^d(k) \circ x^d(k+1)$, may be constructed as an a priori desired trend. Rewrite (3.17) as:

$$\mathcal{P} = f_d(x_k) \wedge g(x_k, u_k) \tag{3.18}$$

where

$$\begin{aligned}f_d(x_k) &= x_d(k+1) \circ x_d(k) = \bigvee_{i=1}^{s} [x^{di}(k+1) \wedge x^{di}(k)], \\ g &= u(k) \circ x^d(k) = \bigvee_{i=1}^{s} [u^i(k) \wedge x^{di}(k)]\end{aligned} \tag{3.19}$$

The system evolution may now be described as:

$$x(k+1) = f_d \circ x(k) \wedge g \circ u(k) \tag{3.20}$$

Expression (3.20) will be referred to as a *fuzzy companion form.*

Solution of the Fuzzy Relational Equation

Let us perform a couple of iterations of this expression, for k = 0 and k = 1.

k = 0 gives:

$$\begin{aligned} x_1 &= f_d \circ x_0 \wedge g \circ u_0, \text{ and for } k=1, \\ x_2 &= f_d \circ x_1 \wedge g \circ u_1 \\ &= f \circ [f \circ x_0 \wedge g \circ u_0] \wedge g \circ u_1 \\ &= [f^2 \circ x_0 \wedge f \circ g \circ u_0 \wedge g \circ u_1] \end{aligned} \tag{3.21}$$

Similarly, k = 2 gives:

$$x_3 = [f^3 \circ x_0 \wedge f^2 \circ g \circ u_0 \wedge f \circ g \circ u_1 \wedge g \circ u_2] \tag{3.22}$$

and for k = n,

$$x_{n+1} = [f^{n+1} \circ x_0 \wedge f^n \circ g \circ u_0 \wedge f^{n-1} \circ g \circ u_1 \wedge ,\dots, \wedge g \circ u_n] \tag{3.23}$$

Using a dummy variable τ, and rearranging, the above is expressed in a short-hand form as:

$$x_\tau = f^\tau \circ x_0 \wedge \left[g \ \ (f \circ g) \ \ (f^2 \circ g) \ \ (f^3 \circ g) \ \ \cdots \ \ (f^{\tau-1} \circ g)\right] \circ \begin{bmatrix} u_{\tau-1} \\ u_{\tau-2} \\ \vdots \\ u_1 \\ u_0 \end{bmatrix} \tag{3.24}$$

Proposition 3.2:

The system in fuzzy companion form (3.20) is *completely controllable*, that is, it can be transferred from the initial state x_0 to any arbitrary state x_k, $k>0$, by the control sequence $\{ u_0 \ u_1 \ \ldots \ u_k\}$, $k = s$, if: i). f and g exist, ii). f o g is nonvanishing and iii). each of the k times composition of f with itself is nonvanishing.

<u>Remark</u>:

The condition stated above is only sufficient, for the companion form being sought may not be realizable and yet the system may be controllable. The conditions may be easy to check when the number of rules and linguistic terms are small. Note that the number of linguistic terms can be related to the cardinality of the fuzzy sets or membership functions. Knowing this, a rank condition similar to the one for the controllability matrix in LTI systems may be imposed.

3.6 Summary

The chapter started by providing an insight into a fuzzy logic control systematic design for stability. The class of systems that can be designed by this algorithmic methodology was identified. It was shown that the algorithmic nature of the design methodology leads to an a priori assumption of stability which needed to be formally verified. Such a verification was considered necessary before the system might be put through any rigorous practical testing. A method for verifying the stability convergence properties of such a rulebase was outlined from a topological view point. Another desirable property closely linked to stability is controllability. A theory was developed for controllability of a special class of fuzzy dynamical systems that were designed strictly from fuzzy rules and fuzzy relations. A set of sufficient conditions for controllability are derived. Finally, a detail of an application example of the systematic design methodology was considered in the design of a fuzzy controller for a model of an automotive engine operating under idle conditions; this was placed in Appendix IV. There, the stability convergence properties were shown on the simulations.

CHAPTER IV

GENERAL INPUT-OUTPUT STABILITY FOR DISSIPATIVE FUZZY CONTROLLED SYSTEMS

4.1 Introduction

This chapter develops a method of input-output stability analysis on the basis of the rules, and the control input values generated by the fuzzy controller for each cell partition of the phase plane. It is assumed that the rulebase has been designed for a linear or nonlinear plant whose approximate model is available. Furthermore, the rules may have been derived using the operator manual-type if-then rules or based directly on the approximate model. In either case, the rules can be displayed either linguistically or numerically, on the phase plane of interest. The method is based on formulating an input-output dissapative map and then applying a Lyapunov-like analysis for stability convergence. For nonlinear systems, there is no direct or unique way of proving the dissipativeness assumption. However an input-output mapping can always be formulated as an energy-like function. By proper choice of outputs, and the inputs generated by the fuzzy controller, the energy-like function is forced to be dissipative. This then satisfies a condition for asymptotic stability. On the other hand, if the fuzzy controller is designed for a linear plant, a similar dissipative mapping is shown to be obtained fairly easily by applying the Kalman-Yakubovich lemma. By so doing, the output selection matrix may have to be restructured; this is shown with an example. Even if the lemma cannot be applied directly, it is a known fact that the linearized model of a stable nonlinear system should be stable in the neighborhood of the equilibrium point. Thus, this allows the Kalman-Yakubovich lemma to be applied in this neighborhood, in the formation of a dissipative input-output map. The direction taken in this chapter is to first determine the stability convergence of the nonlinear system and then demonstrate the proof of dissipativeness in the neighborhood of the origin. Depending on the explicit expression for the time derivative of the Lyapunov-like function for the nonlinear plant, it may be tedious to attempt to determine its definiteness analytically, for arbitrary points, without making undue approximations. Thus, subsets of the rule base will have to be considered in each of the four quadrants of the state space. By choosing an arbitrary point in each quadrant, with its generated fuzzy controls, the time derivative is simulated and conclusions about the definiteness are drawn from the simulation results. Still, there are other remaining requirements that may need to be satisfied before a conclusion about stability can be made.

4.2 Remarks

The difficulty in studying the stability of fuzzy controlled systems is really not due to the lack of analytical tools, but rather, to the fact that a model of the plant, even if only approximate, has not been employed in the design of the fuzzy controller. Nevertheless, the stability of the closed-loop system can be studied by applying the fuzzily derived rules on an approximate model of the plant. The major difficulty arises when all one has is a set of if-then rules with no mathematical model to apply it on for stability analysis. In such cases, one can only look at the phase plane and ascertain what rules are to be fired in any given region, and even determine the range of numerical values of the appropriate control actions. But where a region is left blank to signify, perhaps, an unstable region, there is really not a whole lot that can be said, especially analytically, about the stability of the closed-loop system. There exists, nonetheless, a host of effective techniques for studying the stability of fuzzy dynamical systems based on fuzzy relations alone. The energetistic stability of Kizka et al [8], for instance, offers a very effective technique when the number of rules and cardinality of the fuzzy sets are small. Using this, such dynamical properties as periodic motion, stability and instability can be studied. But then, this method does not use, explicitly, the concept of the fuzzily derived control rules that are actually used to control the physical process. Other methods actually derived a control surface by applying classical stability theorems on the plant model. Later on, elements of the control surface were fuzzified to serve as fuzzy controls [20]. While this is a valid derivation of fuzzy control rules, it falls into the claim of only verifying what the designer has assumed a priori at the design stage [19]. Needless to say, this assumption is legitimate and indeed warranted. It is believed in this research that by whatever means the fuzzy control rules have been derived, the intent is to control a given physical process in closed-loop, so as to maintain a stable operation in addition to meeting certain design objectives. Thus, in the proceeding, it is assumed that a) The mathematical model of the plant is available, even if in an approximate form, b) An explicit form of the control law can be derived either linguistically, numerically or as a function of the process variables and c) The rule base is assumed to be complete. The purpose of (a) above is so that (b) can be applied on the closed-loop system, by formulating stability theorems in a control theoretic sense. To begin, certain terminology will be introduced here which will be used for throughout the thesis.

4.3 The Class of Fuzzy Controlled Plants

The class of fuzzy controlled systems considered is that for which an approximate mathematical model is available and can be expressed generally by:

$$\dot{x} = f(x,u,d),\ x \in \mathbb{R}^n,\ u \in \mathbb{R}^m,\ d \in \mathbb{R}\ \textit{and}\ f\colon \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^n. \quad (4.1)$$

f is generally a nonlinear mapping, u is the control input which is bounded in a known universe of discourse, x is the state with a specified universe of discourse, and d is an additive scalar disturbance of known origin and bounded magnitude.

Define: F(U) $\subset$ $\mathbf{R}^*$, the set of feasible inputs, and F(X) $\subset$ $\mathbf{R}^*$, the set of feasible states. $\mathbf{R}^*$ is the set of fuzzy numbers.

Let the fuzzy control rule be of the form:

Rule *i* : If e_1 is Le_1 and e_2 is Le_2 ... and e_n is Le_n Then
u_1 is Lu_1 and u_2 is Lu_2 ... and u_m is Lu_m

where Le_i and Lu_i are the linguistic terms for the process variables. These variables are often the state errors or the actual states on the premise side and the process control inputs on the consequent side. The crisp values u_j, j = 1, m, are the defuzzified control inputs needed to drive the process, f. For the purpose of illustration, consider the premise vector $x \in \mathbf{R}^2$, and the consequent vector $u \in \mathbf{R}^2$.

The fuzzy control rule base for the generality of the class of systems considered in the research is given as in either case of Figure 4.1.

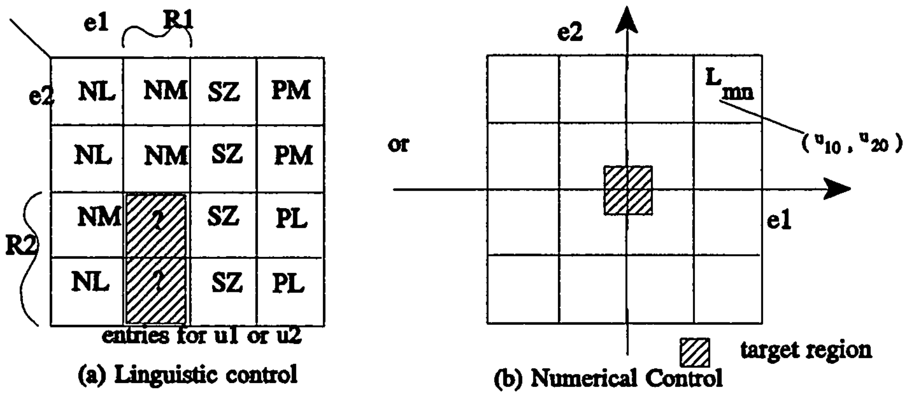


Figure 4.1 Fuzzy Controls Representation

Figure 4.1(a) is very common in an operator manual-type fuzzy logic control rule representation, where the control entries are labels such as negative large (NL), negative medium (NM), small near zero (SZ), positive large (PL), positive medium (PM), etc. These are the linguistic control values for u_1 or u_2. Figure 4.1(b) on the other hand, gives a more elaborate representation. Here, the actual, crisp control sequence values are placed in each region of the phase plane, which is commonly taken as the "error" space. For example, for the case of 2 inputs, the region or cell L_{mn} has, $u_1 = u_{10}$ and $u_2 = u_{20}$, which are displayed simultaneously on the cell as (u_{10}, u_{20}). This method therefore allows for easy representation of more than one control input simultaneously.

4.4 Rulebase Completion

Definition 4.1: A rulebase is *complete* if its phase plane representation does not contain any *gap*.

Definition 4.2: A *gap* is said to exist in the case of Figure 4.1(a) for example, in the regions denoted by "?", or where a region is left void of any control rule, linguistic or otherwise.

Existence of a Gap

A Gap can exist for a variety of reasons. (i) Suppose for some $\forall u \in F(U)$, $f(x, u) \Rightarrow x \notin F(X)$, then a case of nonfeasible or divergent states results. (ii) Similarly, suppose for some $x(t_1) \in F(X)$, $\nexists\ u \in F(U) \ni f(x(t_0),u) = x(t_1)$, $t_0 < t_1$, then such $x(t_1)$ is considered *uncontrollable* and therefore *inaccessible*. In either case, a gap may result. It may be possible at times to "extend" the domains so that these properties hold. For example, in Figure 4.1(a), a new set of feasible states may be defined as $F(X)\backslash (R_1 \times R_2) = F_1(X)$, and $F_1(X)$ will be complete, and will therefore be controllable. However, care must be exercised not to deform the domain to the extent that a usable portion of the physical operation space of the system is removed. One way to achieve this is to construct a rule that will dump any $R_1 \times R_2$ to a sink region at infinity, for all future time, whenever $(e_1,e_2) \in R_1 \times R_2$. This is preferable to leaving such a region blank. The resulting space will therefore be an invariant set. It is not difficult to argue that a complete rulebase will always be found for such an invariant set. A corollary to this is that, any point in the invariant set is both accessible and controllable. Accessibility is defined below.

Definition 4.3: A *Reachable set* at time k for the system with initial condition $x(0) = x_0$ is defined as the set of all states $x_1 \in F(X)$ such that there exists $u \in F(U)$ on $[0, k] \ni f(x(0), u) = x_1$. Denote this set by $R(k, x_0)$. The reachable set is then given by:

$$R(x_0) \triangleq \bigcup_{k \geq 0} R(k, x_0) \tag{4.2}$$

Definition 4.4: The system is said to be *accessible* from x_0 if $R(x_0)$ contains a neighborhood of some $x \in F(X)$, that is, if $R(x_0)$ has a non empty interior.

The system is accessible from each $x_0 \in F(X)$ and controllable if $R(x_0) = F(X)$, $\forall x_0 \in F(X)$. Note that a system can be accessible and yet not be controllable [72]. Nonlinear systems are notorious for exhibiting these characteristics. A detailed treatment of this issue can be found in [63,79]. It suffices to say that it is an important point to remember when dealing with nonlinear systems.

4.5 Center-point Center-control Reference

From the representation in Figures 4.1, the "control law" may take the form:

u = (NL, NM, SZ, PM,...) or for $u \in \mathbb{R}^2$, $u = (u_{10}, u_{20})$.

The representations are not mutually exclusive. The linguistic form, for instance, can be defuzzified, given an incoming data for the premise variables, to obtain a crisp representation for the particular region. The meaning of the second case above with the crisp values (u_{10}, u_{20}), is that a snap shot is taken at given incoming data (x_{10},x_{20}) into the fuzzy controller. The control values corresponding to this data after defuzzification are therefore the crisp values labeled, (u_{10},u_{20}). Based on the general design methodology discussed thus far, what immediately comes to mind is that what control value should be used when the states fall into a given cell, and what point in the cell should be referred to for transitioning to the next cell. Much of this is still a research issue. A particular point, $(x_1\ x_2)$, belonging to one or more linguistic sets at an instant time i, may equivalently be described as belonging to a specific cell as given below.

$$if\ x_{1\min} \leq x_1 \leq x_{1\max}\ \ and\ \ x_{2\min} \leq x_2 \leq x_{2\max}\ \ THEN\ \ (x_1,x_2) \in L_{mn} \quad (4.3)$$

This is essentially the fuzzy rule for the cell labeled as L_{mn}. The center-point is suggested to be a cell's representative point in this thesis. This point is therefore used for control application and transitioning into the next cell. This representation is depicted in Figure 4.2 below.

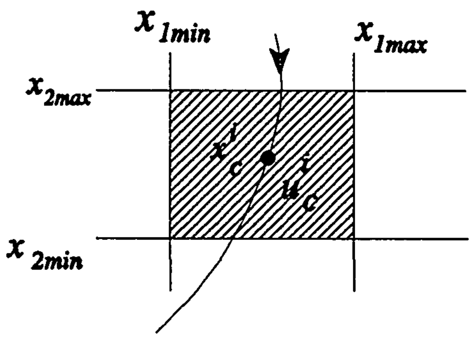


Figure 4.2 Center-point Center-control

In general, let the control law be represented by $u_1 = g_1(e), \ldots, u_m = g_m(e)$ where u_i, $i=1, \ldots, m \in F(U)$ have been obtained via defuzzification.

4.6 Stability of Nonlinear Fuzzy Control Systems:

Method I

It is required to study the asymptotic stability convergence of the closed-loop system with the fuzzy control inputs being continually generated by the fuzzy controller in the system. The convergence point is a specified set-point.

Assumption: The system is assumed to be dissipative so that one can argue heuristically, that the total energy of the system decreases as time progresses, until a state of equilibrium is reached.

Lemma 4.1: If the differentiable function $V(t)$ has a finite limit as $t \to \infty$, and if $\dot{V}(t)$ is uniformly continuous, then $\dot{V}(t) \to 0$ as $t \to \infty$.

This is a statement of Barbalat's lemma. However, the class of systems considered here is autonomous, so the uniform continuity restriction may be alleviated. The consequence of this is the following Lyapunov-like lemma.

Lyapunov-Like Lemma 4.2: Let the scalar function $V(x,u)$ satisfies the following conditions:

(1) $V(x,u)$ is lower bounded

(2) $\dot{V}(x,u)$ is negative semi-definite

(3) $\dot{V}(x,u)$ is uniformly continuous in u

then $\dot{V}(x,u) \to 0$ as $t \to \infty$

Note that u in above lemma are the fuzzy control inputs that are being generated as the system evolves. These control inputs are used to induce the input-output dissipative mapping in the expression 2 in Lemma 4.2 above, for all cells in the rulebase. It seems, however, that the continuity of x under u needs to be assured so that *V* and its time derivative will be continuous in x under u. This is proved in the following.

(Proof): Continuity of x(t) under fuzzy control u

Consider the center-point, center-control method already introduced. The center control u_c is defined to be uniform for the particular $x \in L_{mn}$. Thus, for some x^*, the $\|x^* - x_c\| \leq \delta$, $\delta \neq \varnothing$. Let the control in this set be u^*. For δ small, this implies that $u^* \in B(u_c, \varepsilon)$, for ε small. By the center-control construction, $u^* \equiv u_c$. This is therefore a form of smoothness of x(t) with respect to the fuzzy control u■

The next thing to do is find a suitable *V(x)* from which the dissipative mapping may be formed. Since it is assumed that the fuzzy controller is stable, a converse stability theorem assures the existence of a Lyapunov function.

Theorem 4.1: If the origin of (4.1) is stable, then there exists a positive definite function *V(x)* with a non-positive derivative.

4.6.1 Equilibrium Points for Fuzzy Controlled Processes

In classical Lyapunov stability of system of the type in (4.1), the unforced system $f(x_e, 0)$ is studied with $f(x_e, 0) = 0$, where x_e is the equilibrium state. $x_e = 0$ is often employed with, actually, some loss of generality.

Some Technicalities:

1. In a variety of physical processes, the equilibrium point $x_e = 0$ may not be convenient or even feasible. For example, an aircraft cruising at a certain speed at a given cruising altitude can be considered as a steady state condition. Clearly, the throttle, and the vertical stabilizer among other control surfaces, cannot be set to zero in order to study the stability of the cruise controller. In such a case, it is appropriate to define the equilibrium point as the error $e_1 = x_1 - x_{1d}$, and $e_2 = x_2 - x_{2d}$, where x_1 and x_2 may be the speed and the vertical stabilizer angle, in which case $e_1 = e_2 = 0$ is not only feasible but has the proper physical significance as well. In general, one can always define a new equilibrium point via a shift of origin. This freedom should always be exercised when dealing with a real, physical process.

2. It is not always practicable to remove the "forcing" function from a physical process that is in operation, and expect it to even function at all, let alone satisfactorily. Thus, the study of $f(x_e, u)$ as $f(x_e, 0)$, with $u = 0$, is not always a physical reality as can be imagined from the aircraft scenario given above. Also, in a variety of physical devices, certain nonzero *bias* inputs are always needed to set the device in a proper operational regime. Take the case of a bipolar junction transistor, for instance. It would be inconceivable to attempt to use it as an amplifier, without the required bias currents or voltages. There is a host of engineered processes that one can think of that command this particular kind of attention for their steady state input-output stability analyses. This is why this particular method of stability analysis was proposed. In light of this, the stability of the fuzzy control process will be studied with $u \neq 0$ in the various regions of the phase plane, unless the fuzzy controller explicitly suggests such a zero value in a particular region. For linear fuzzy controlled systems, this zero value of the control will be shown to cause some difficulty in the analysis, when $x_e = x_d \neq 0$.

4.6.2 The Dissipative Input-Output Mapping

Let $V(x)$ be such a lower bounded scalar function. Note that this is what makes this only a Lyapunov-like analysis as $V(x)$ is not required to be positive definite. It is given by:

$$\dot{V} = \frac{\partial V}{\partial e}\dot{e} = \frac{\partial V}{\partial e} f(x,u) \quad with\ e \triangleq x - x_d, \quad x_d \in \mathbb{R}^n. \tag{4.4}$$

If y = g(x) is the output expression, the x = $g^{-1}(y)$, then the output-input map becomes:

$$\dot{V}(g^{-1}(y),\tilde{u}) = \frac{\partial V}{\partial e}\dot{e} = \frac{\partial V}{\partial e}\Big|_{(x_d,\,\tilde{u})} f(g^{-1}(y),u). \tag{4.5}$$

where ũ are the fuzzy control inputs. To be dissipative is to satisfy condition 2 in lemma 4.2 in all cells with their respective fuzzy controls. For a very complex expression, it may be convenient to simulate this expression for various cells in the phase plane in order to determine the definiteness. Finally condition 3 of lemma 4.2 needs to be checked. This can be done in a less cumbersome way by showing that d^2V/dt^2 is bounded. The conclusion of the lemma then follows. If a region or cell of the phase plane can be found, with its associated fuzzy control, for which dV/dt fails to be negative semi-definite, ∀t, then the fuzzy control system is unstable. For this particular region, it would be possible to track the fuzzy control down to the membership function level and redesign the particular rule in question, rather than discard the entire rulebase. As observed, the practicality of this analysis is apparently limited when: 1). The number of regions or cells in the phase plane is large and there are different controls for each cell. 2). The expression for the nonlinear process is very complex, and renders any analytical determination of the definiteness of dV/dt near impossible.

One way to circumvent this as suggested above, is to actually simulate $dV/dt|_{\alpha}$ for various points in the four quadrants of the phase plane. Another way is to prudently cluster the rulebase into a fewer number of rules; this was done for the test-bed fuzzy controller in Chapter VI, where the number of rules was reduced from 56 to 7 via the process of clustering. In general, to be complete, this simulation will have to be exhaustive, especially for a suspicious, highly nonlinear and potentially unstable process.

4.7 Stability of Linear Fuzzy Controlled Systems

Suppose the fuzzy controller is designed to control a linear plant with a transfer function G(s) or state space elements $\Sigma(A,B,C,D)$. The system is normally expressed as:

$$\sum : \dot{x} = Ax + Bu, \quad y = Cx + Du. \tag{4.6}$$
$$with\ x \in \mathbb{R}^n, A \in \mathbb{R}^{nxn}, B \in \mathbb{R}^{m\times n}\ u \in \mathbb{R}^m, C \in \mathbb{R}^{r\times n},\ D \in \mathbb{R}^{r\times m}.$$

The block diagram of the closed-loop fuzzy control system is shown in Figure 4.3 below.

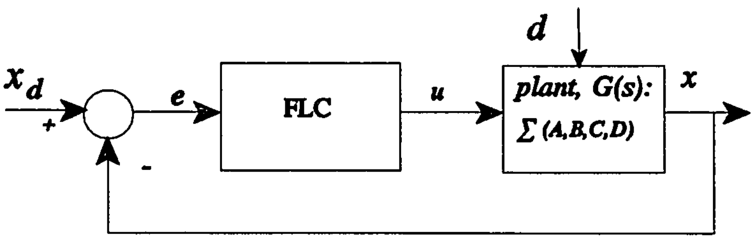


Figure 4.3 Linear Fuzzy Control System

In this case, an input-output map can be formed relatively easily. Systems with no direct feed are considered so that D = 0 in (4.6). Suppose the system is opened-loop stable. A converse stability theorem assures of the existence of a Lyapunov function, that fulfills condition 1 of Lemma 4.2. Let this scalar function be $V(x) = 1/2\ x^T P\ x$, where P is a positive definite symmetric matrix. This assures that *V(x)* is also lower bounded. The time derivative is given as:

$$\frac{dV}{dt} = \frac{1}{2}\left[\dot{x}^T P x + x^T P \dot{x}\right] = \frac{1}{2}\left[(x^T A^T + u^T B^T) P x + x^T P A x + x^T P B u\right]$$

$$= \frac{1}{2} x^T (A^T P + PA) x + \frac{1}{2}(x^T PBU + x^T PBu) \quad (4.7)$$

$$= \frac{1}{2} x^T (A^T P + PA) x + x^T PBu$$

For global stability, $dV/dt < 0$. The first term can be made negative definite by making A stable independent of B and C, by setting $A^T P + PA = -Q$, for some positive definite symmetric matrix, Q. Doing this results in:

$$\frac{dV}{dt} = -\frac{1}{2} x^T Q x + x^T PBu, \quad -Q = A^T P + PA. \quad (4.8)$$

$$\frac{dV}{dt} < 0 \Rightarrow \frac{x^T Q x}{x^T x} > \frac{2 x^T PBu}{x^T x} \quad (4.9)$$

Note that the left hand side of the implied inequality in (4.9) is actually Rayleigh's quotient, which can be derived from the eigenvalue equation for a matrix Q that is real and symmetric. If the eigenvalues of Q are numbered such that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$, then

$$\lambda_n \leq \frac{x^T Q x}{x^T x} \leq \lambda_1, \quad r = \frac{x^T Q x}{x^T x} = Rayleigh's\ Quotient. \quad (4.10)$$

then $\lambda_1 \geq r$, $\lambda_n \leq r$ where $\lambda_n \equiv \lambda_{min}$, and $\lambda_1 \equiv \lambda_{max}$ [84].

As a consequence of this,

$$\lambda_{\min}(Q) > 2\frac{x^T PBu}{\|x\|^2}, \quad Q = A^T P + PA, \ x \in L_{mn} \ and \ u = u_c. \quad (4.11)$$

Global Stability: If (4.11) holds in every region, L_{mn}, of the state space, then the system is globally, asymptotically stable. An analysis similar to this cited in the literature stopped here [55]. To summarize, the method lacked generality. Therein, minimum and maximum control bounds were determined for each region in the state space. The center-point, center-control method was not used and this complicated the analysis as to the selection of representative control and representative points while in a given cell. Also, the assumption of dissipativeness was not verified, as will be done here shortly.

Suppose, as often assumed in the thesis, that the desired equilibrium point is $x^* = x_d$ is other than the origin, $x = \mathbf{0}$, then the trajectory error $e = x - x_d$ can be formed such that the state equation (4.6) becomes the error system below.

$$\dot{e} = Ae + Ax_d + Bu. \tag{4.12}$$

Given x_d, the term Ax_d may be considered as an offset that the fuzzy controller must overcome for all L_{mn}. It will be required therefore that $Ax_d \in \rho(B)$, the range of B, $\forall u$ and $x_d \neq 0$. That is, $\exists\, B_{mn} \neq 0 \ni Ax_d = B_{mn} u_c$. This will be satisfied for $u_c \neq 0$. The trivial case when $x_d = 0, u = 0$ is not particularly interesting in the development of an input-output stability, for it refers to the unforced system $\dot{x} = Ax$ for which $x = 0$ is the only equilibrium point. With such B_{mn}, the error system becomes:

$$\dot{e} = Ae + (B_{mn} + B)u = Ae + B'u \tag{4.13}$$

The same analysis can now be carried out with a broader generality.

4.7.1 Verifying Dissipativeness

Consider equation (4.8) which is reproduced below for convenience.

$$\frac{dV}{dt} = x^T PBu - \frac{1}{2} x^T Qx \tag{4.14}$$

A variation of the Kalman-Yakubovich lemma is stated below for a controllable system.

Lemma 4.4 : Consider the controllable linear time-invariant system, $\Sigma: \dot{x} = Ax + Bu$, *with transfer function* $G(s) := C^T(sI - A)^{-1}B$, *and* $y = C^T x$. $G(s)$ *is strictly positive real if and only if there exist positive definite symmetric matrices P, Q such that* $AP + PA = -Q$, *and* $PB = C$.

Alternatively, since $y = Cx$, (4.14) defines a dissipative mapping between u and y provided that B and C are related by $C^T = PB$. So, $y^T = x^T C^T = x^T PB$ and equation (4.14) becomes:

$$\frac{dV}{dt} = y^T u - \frac{1}{2} x^T Qx \tag{4.15}$$

Thus, by compatible choices of u and y, a family of dissipative input-output maps can be constructed. That is, given the system's fuzzy control inputs and the input distribution matrix, B, one can choose an infinity of outputs from which the linear system will appear dissipative. If expression (4.15) above fulfills condition 2 in Lemma 4.2 for compatible choices of u and y, the dissipativeness of the system is therefore verified.

4.8 Applications

In this section, two application examples are considered. The first one is the test-bed fuzzy logic controller for the nonlinear automotive engine. The second example considers the fuzzy control of a missile autopilot. These fuzzy logic control system have been designed here at Georgia Tech using the systematic design methodology outlined in Chapter III and dtailed in Appendix IV. Only the relevant results are reproduced here.

4.8.1 Nonlinear Example: Engine Idle Speed FLC

The test-bed fuzzy controller for the automotive engine idle speed is employed as an example. Details of the model can be found in Appendix IV. The dynamics is given by:

$$\begin{aligned} \dot{P} &= k_p(\dot{m}_{ai} - \dot{m}_{ao}) \\ \dot{N} &= k_N(T_i - T_L) \end{aligned} \qquad (4.16)$$

The state vector is defined as $\mathbf{x} \in \mathbb{R}^2$, and the control vector $\mathbf{u} \in \mathbb{R}^2$, where

$$\boldsymbol{x} = (x_1 \;\; x_2)^T \triangleq (P \;\; N)^T \; , \quad \boldsymbol{u} = (u_1 \;\; u_2)^T \triangleq (\theta \;\; \delta)^T \qquad (4.17)$$

The errors in P and N are defined as:

$$e_1 \triangleq P - P_0 = x_1 - P_0, \; and \; e_2 \triangleq N - N_0 = x_2 - N_0 \qquad (4.18)$$

<u>System Transformation</u>.

For the purpose of the stability analysis, let us transform the model into the form (4.1) under the state and control vector definitions (4.17) and (4.18). Upon doing this, the dynamics become:

$$\begin{aligned} \dot{x}_1 &= k_p(\dot{m}_{ai} - \dot{m}_{ao}) \\ \dot{x}_2 &= k_N(T_i - T_L) \end{aligned} \tag{4.19}$$

where,

$$\begin{aligned} \dot{m}_{ai} &= (1 + 0.907u_1 + 0.0998u_1^2)g(x_1) \\ \dot{m}_{ao} &= -0.0005968x_2 - 0.1336x_1 + 0.0005341x_2x_1 + 0.000001757x_2x_1^2 \\ T_i &= -39.22 + \frac{325024}{120x_2}\dot{m}_{ao} - 0.0112u_2^2 + 0.000675u_2x_2(2\pi/60) \\ &\quad + 0.635u_2 + 0.0216x_2(2\pi/60) - 0.000102x_2^2(2\pi/60)^2 \\ T_L &= (x_2/263.17)^2 + T_d \\ g(x_1) &= \begin{cases} 1 & x_1 < 50.66 \\ 0.0197(101.325x_1 - x_1^2)^{\frac{1}{2}} & x_1 \geq 50.66 \end{cases} \end{aligned} \tag{4.20}$$

$$\dot{x}_1 = k_p[a_0x_2 + a_1x_1 - a_2x_1x_2 - a_3x_2x_1^2 + g(x_1)a_4] + [k_pg(x_1)a_5]u_1 + k_p[g(x_1)a_6u_1^2]$$

$$\triangleq F_1(x,u,T_d)$$

$$\dot{x}_2 = k_N[-a_7 + a_8\frac{\dot{m}_{ao}}{x_2} + a_{12}x_2 - (a_{13}+a_{14})x_2^2] + [a_{10}x_2 + a_{11}]k_Nu_2 + [-a_9k_N]u_2^2 + a_{15}T_d]$$

$$\triangleq F_2(x_2,u,T_d) \tag{4.21}$$

where the coefficients, a_i, and their values are given below.

List of constants

$a_0 = 0.0005968$	$a_1 = 0.1336$	$a_2 = 0.0005341$
$a_3 = 0.000001757$	$a_4 = 1$	$a_5 = 0.907$
$a_6 = 0.0998$	$a_7 = 39.22$	$a_8 = 2708.533$
$a_9 = 0.0112$	$a_{10} = 0.000070686$	$a_{11} = 0.635$
$a_{12} = 0.002306$	$a_{13} = 0.0000011856$	$a_{14} = (1/263.17)^2$
$a_{15} = -k_N = -54.26$	$k_p = 42.4$	

The Unforced System

Consider $F(x,u, T_d)$. The "unforced" or unloaded system is, $F(x,u,0)$, obtained by setting $T_d = 0$, not $\mathbf{u} = 0$. By the "forced" or loaded system, the same functions are referred to, but with $\Delta T_d \in (0, 61]$ Nm. The Stability of the unloaded system, upon which the fuzzy controller was designed, is studied. It is important that the stability of this no-load rulebase be guaranteed.

Stability Analysis

The error system is formed as $\dot{e} = F(e, u, 0)$, for the unloaded system. The origin, $\mathbf{e} = 0$, is the predetermined equilibrium state of the system. Among all possible scalar functions, let us choose the lower bounded $V(e_1, e_2)$, as:

$$V(e_1,e_2) = \frac{1}{2}e^T e = \frac{1}{2}(e_1^2 + e_2^2) \qquad (4.22)$$

Then,

$$\dot{V}(e_1,e_2) \triangleq \frac{dV}{dt} = e_1\dot{e}_1 + e_2\dot{e}_2 = (x_1 - P_0)F_1(x,u) + (x_2 - N_0)F_2(x,u) \qquad (4.23)$$

with F_1 and F_2 given in (4.13). The explicit forms of these expressions are messy, it will be very tasking indeed to try to show, analytically, that (4.15) is negative definite for general x_1 and x_2 in the invariant set. Instead, representative points will be considered in the quadrants Q1 to Q4 as shown on Figures 4.4 and 4.5, and substitute these points into (4.15), together with the initial and terminal control values as predicted by the fuzzy controller, then simulate the system and conclude from the results. Sample points in these quadrants cover the possible regimes of operation of the engine model, that is, idling at speeds that are above and below the nominal idle speed. In the simulation, the initial control will prevail until the trajectory reaches the vicinity of the target cell, then the control is changed to the predicted fuzzy control of the target cell. This is easily done in a few lines of program.

For the initial point (x_1 = 51 kPa, x_2 =1090 rpm) $\in L_{86}$, one can obtain the sequence of cell-groups traversed by the trajectory in going to the target cell-group, L_{54}, by simply overlaying Figure 4.4 over Figure 4.5. This is observed to be:

$$\underset{U_{11}}{L_{86}} \to L_{85} \to L_{84} \to L_{83} \to L_{73} \to L_{63} \to L_{64} \to \underset{U_{12}}{L_{54}} \qquad (4.24)$$

The FLC predicted that if $(x_1, x_2) \in L_{86}$ then the control U11 $=[\delta_1, \theta_1]$ = [26.1, 4.01] degrees, should produce asymptotic convergence to the target cell, L_{54}. To remain in the target cell for all future time, the predicted control is U12 = [28.1, 6.01] degrees.

So, for each sample point in Q_i, i=1,2,..,4, One needs to show that $\dot{V}(e)$ is negative definite,that is:

$$\frac{dV(e_1,e_2)}{dt}\Big|_{Uij\,then\,U12} < 0,\ \dot{V}=0\ only\ at\ e=0. \tag{4.25}$$

It will be concluded from this and the fact that the Lyapunov function is *coercive*, that is, *V(e)* is *radially unbounded*, that the FLC system is globally asymptotically stable and that **e** = 0 is the only equilibrium state. Globality here refers to the invariant set.

4.8.1.1 Simulation Studies

The simulations have been conducted under a 3-bin, cell-group's control input quantization, $u_{mn} = (\delta_i,\theta_j)$, given below.

$$[\delta_i] = [\delta_1,\delta_2,\delta_3] = [26.1, 28.1, 30.1] \quad and \quad [\theta_j] = [\theta_1,\theta_2,\theta_3] = [4.01, 6.01, 8.01] \tag{4.26}$$

Figure 4.4 shows only the chosen cell-groups at the completion of the search. The automatically generated rules are placed in their respective cell-groups. Observe also at the background of Figure 4.4, the trajectories emanating from various initial center-points as they tend to spiral towards the origin. Finally, Figure 4.5 shows the asymptotic convergence of the FLC for an arbitrary initial condition (Quadrant, Q1) of N = 1090 rpm and P = 51 kPa. The results of the stability analysis are shown on the simulations on Figures 4.5 through 4.9.

Stability Proof:

The results of the stability studies show good stability properties in general. The errors are seen to converge to zero very quickly from all the sample points in every quadrant of the state space.

1. V is lower bounded as chosen.

2. Condition (2) is verified via simulations of dV/dt. Despite the initial transients, dV/dt is decreasing and is predominantly negative and converges to zero in all cases. For all practical purposes, therefore, dV/dt is predominantly negative definite.

3. The continuity of $\partial V/\partial e|_{(xd,\ \tilde{u})} f(x, \tilde{u})$ depends of the continuity of x under $\tilde{u}$, which has been proved in section 4.6.

It is concluded that the system is globally asymptotically stable, and that $\mathbf{e} = 0$ ($e_1 = 0 \Rightarrow x_1 = 34.25$kPa, and $e_2 = 0 \Rightarrow x_2 = 750$ rpm) is the only equilibrium point.

4.8.1.2 Simulation Results

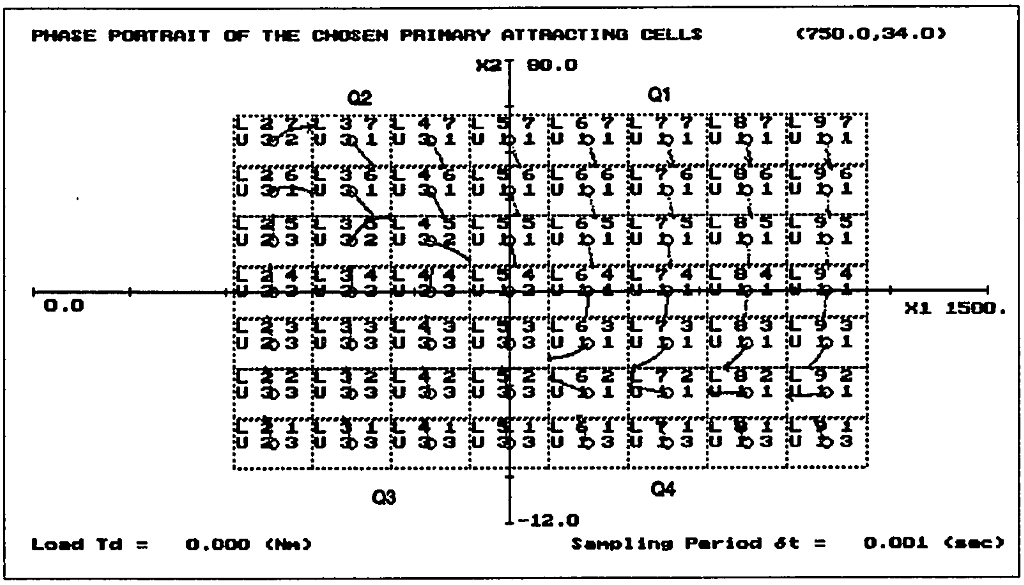


Figure 4.4 The Complete Rulebase

Stability Plots:

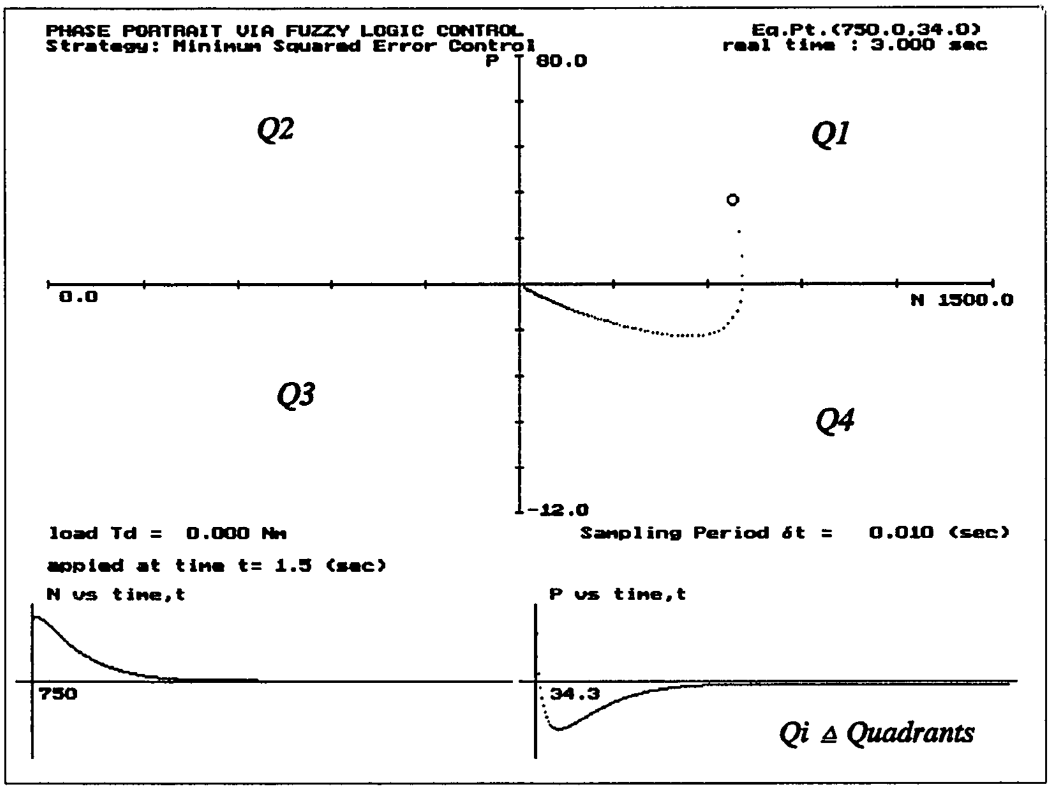


Figure 4.5 Asymptotic Convergence of the Rule Base.

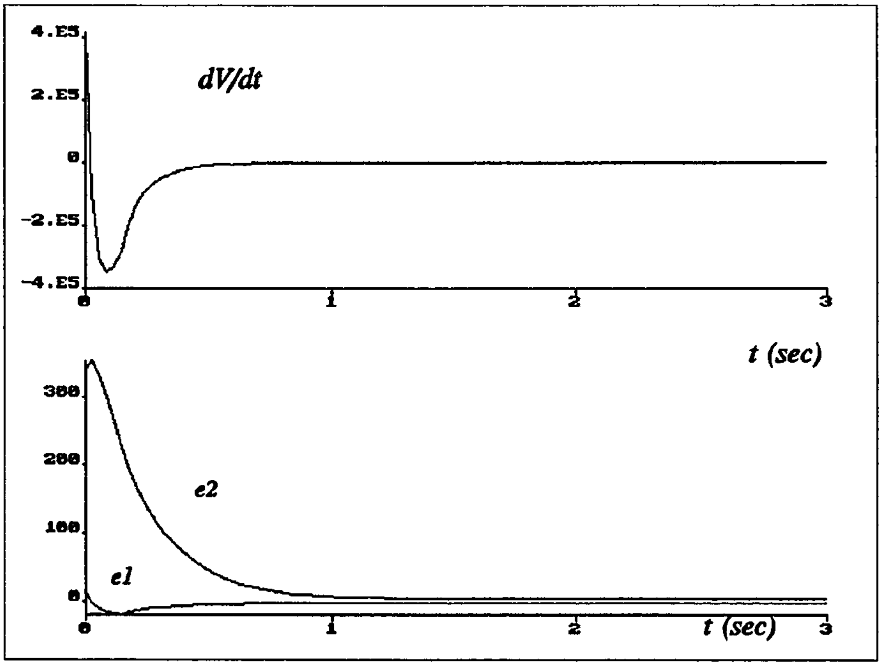


Figure 4.6 Q1: L86,U11, 51kPa, 1090rpm

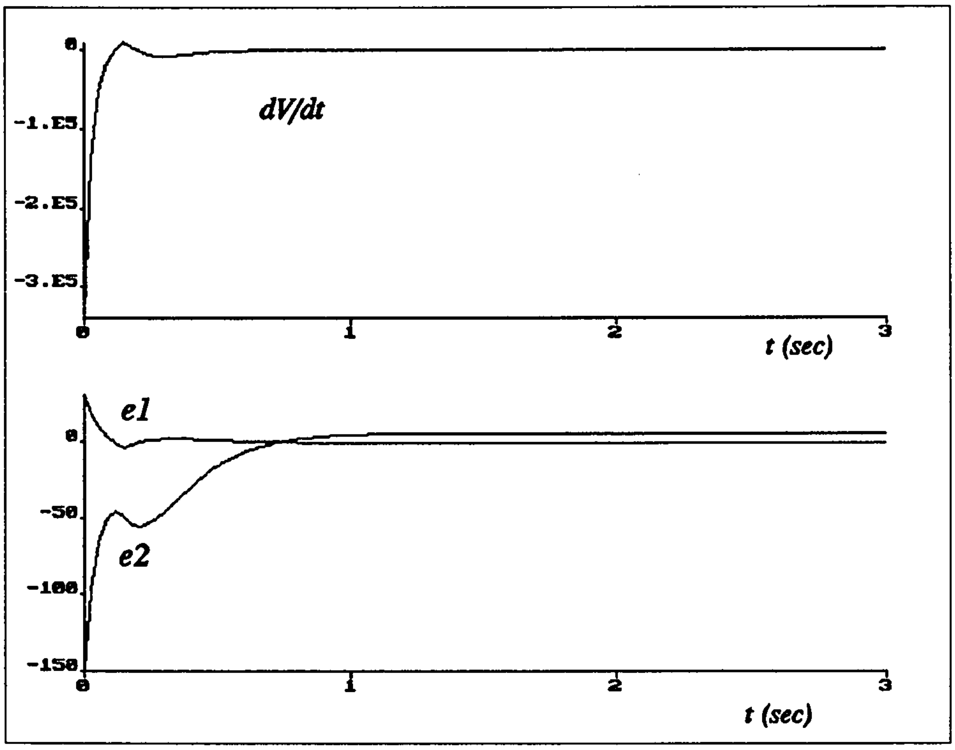


Figure 4.7 Q2: L47,U31, 65kPa, 600rpm

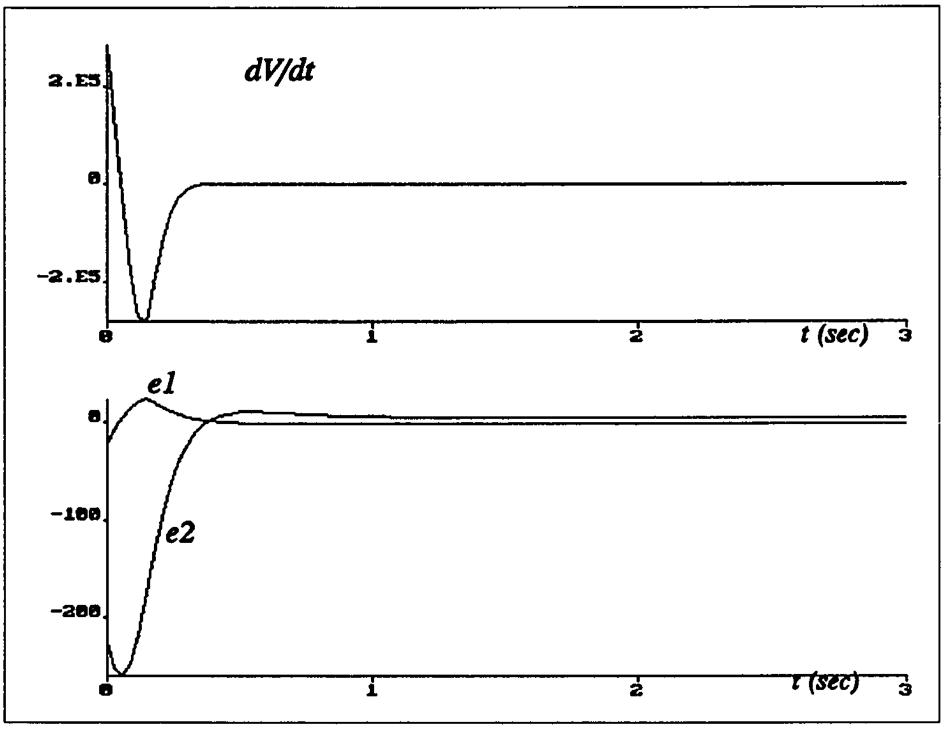


Figure 4.8 Q3: L32, U33, 12kPa, 525 rpm.

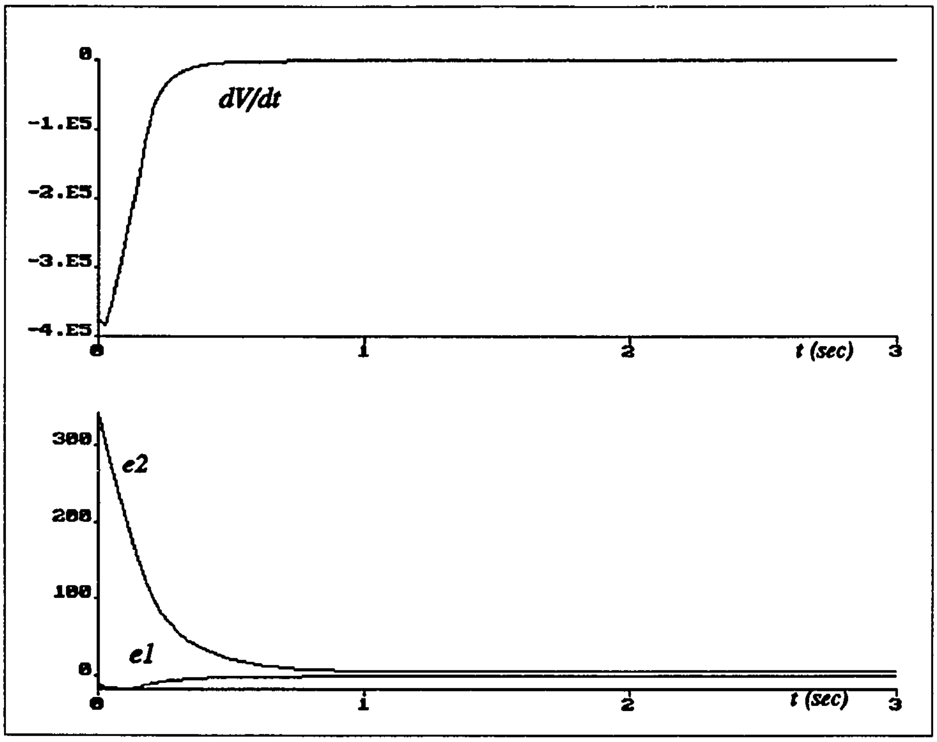


Figure 4.9 Q4: L82,U11, 21 kPa, 1090 rpm.

4.8.2 Linear Example: Missile Autopilot FLC

Consider the regulation problem of a missile autopilot's yaw axis. The linearized dynamic equation of the yaw axis, under consideration, is given by:

$$\begin{bmatrix} \dot{r} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -0.791 & 0 \\ -1 & -0.013 \end{bmatrix} \begin{bmatrix} r \\ \beta \end{bmatrix} + \begin{bmatrix} -1.721 \\ 0.0213 \end{bmatrix} \delta_r \triangleq \dot{x} = Ax + Bu, x = \begin{bmatrix} r \\ \beta \end{bmatrix}, u = \delta_r, \qquad (4.27)$$

where β is the side slip and r is the yaw rate. These are known as the aerodynamic angles. The linearized model was determined on the basis of such quantities as dynamic pressure, Mach number, velocity, and angle of attack. δ_r is the rudder position.

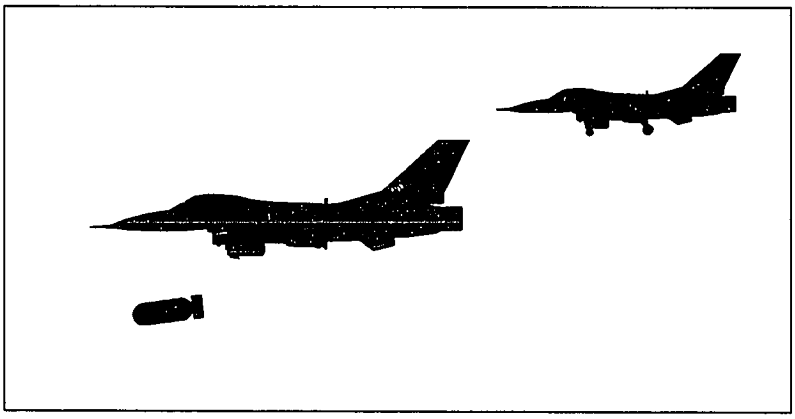


Figure 4.10 A Missile Autopilot's Launch

It is required that the missile's reference point, $\beta = 0, r = 0,$ at the moment of release from the aircraft, be maintained.

The regulation of this yaw axis has been achieved via fuzzy logic control using the systematic design methodology reported in the thesis. The membership functions for r, β and the only control, δ_r are given in Figures 4.11, 4.12 and 4.13, respectively, for illustration purposes. A sample from the 40 fuzzy control rules used to control this system is also shown in Figure 4.14.

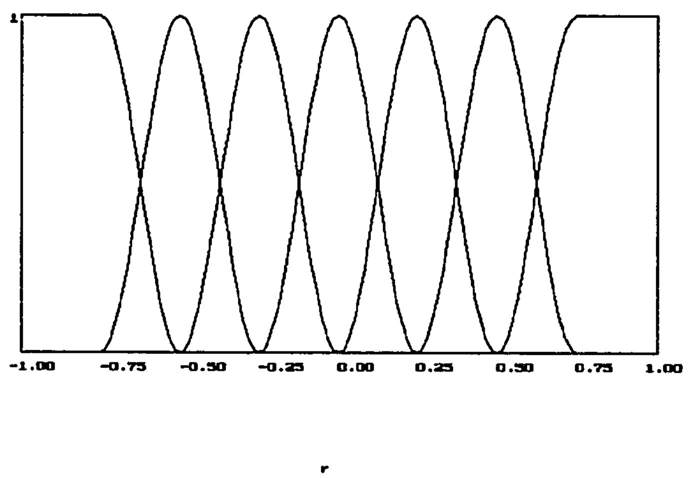


Figure 4.11 Membership Function for Yaw rate (r)

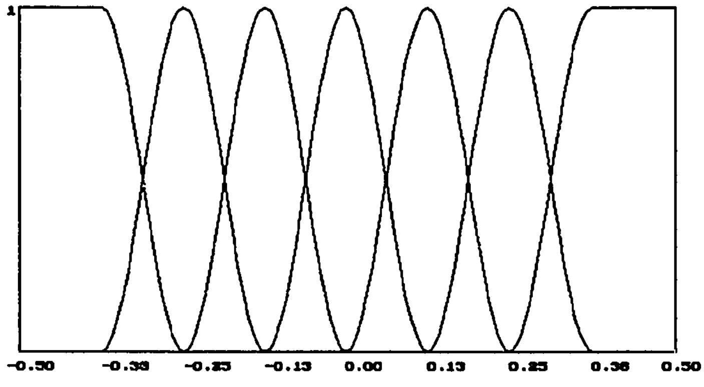


Figure 4.12 Membership Function for Side-Slip β

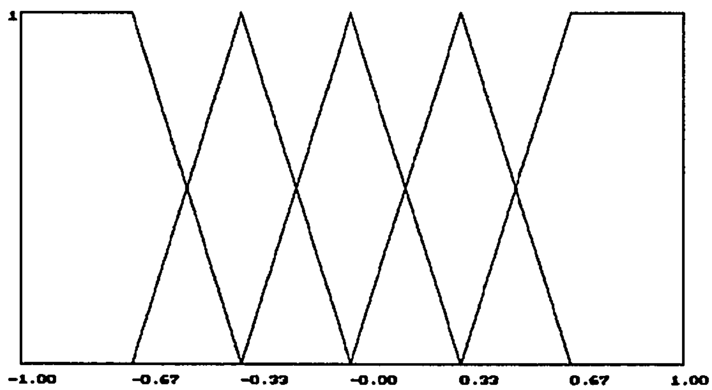


Figure 4.13 Membership Function for Rudder Position (δr)

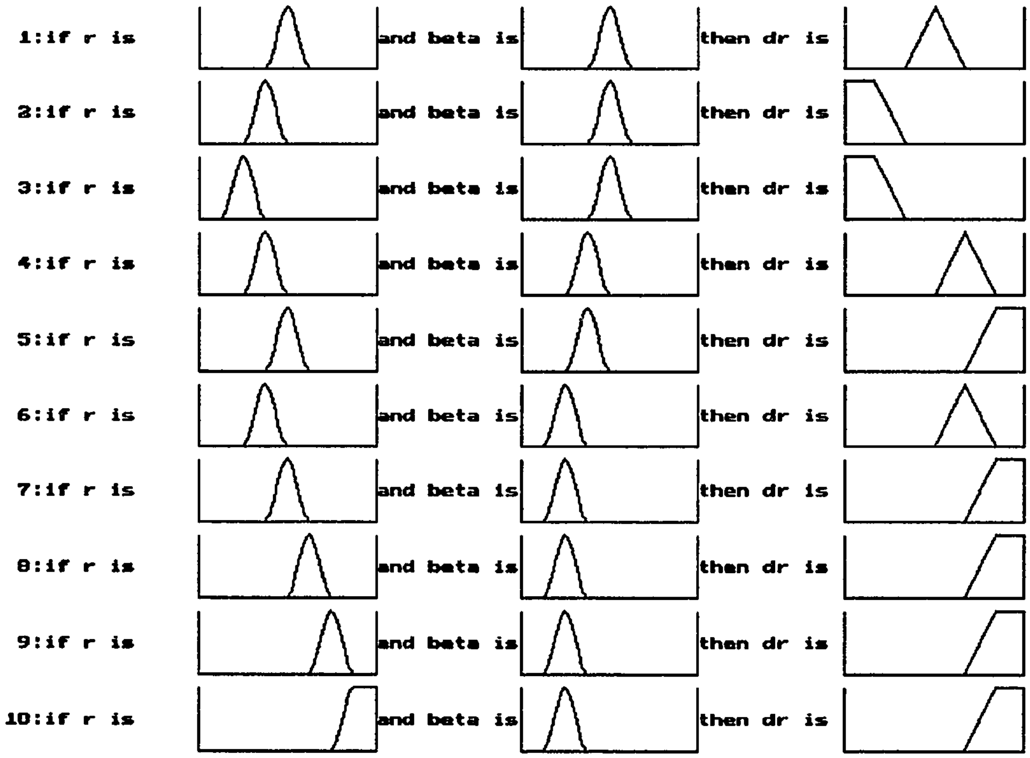


Figure 4.14 Partial Fuzzy Rulebase for the Yaw Axis Controller

Analysis

First, one needs to check that the system is controllable. The controllability matrix, U = [B AB] is found to be full rank, hence it is controllable. A and B are given in (4.27). Let the observed outputs of interest be $y_1 = r = x_1$ and $y_2 = \beta = x_2$. An input-output mapping is then formed as:

$$\frac{dV}{dt} = y^T PBu - \frac{1}{2} x^T Q x \tag{4.28}$$

where P and Q are to be determined. The eigenvalues of A are $\lambda_1(A) = -0.013$, and $\lambda_2(A) = -0.7910$, and so the unforced system is asymptotically stable. By a converse stability theorem, $\exists\ P, Q > 0 \ni A^T P + PA = -Q$. On solving this so-called Lyapunov equation, it is found that:

$$P = \begin{bmatrix} 122.21 & -95.67 \\ -95.67 & 76.92 \end{bmatrix}, \ and \ \ Q = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \tag{4.29}$$

Expression (4.28) then becomes:

$$\frac{dV}{dt} = (-212.3612 y_1 + 166.2865 y_2) u_{FLC} - x_1^2 - x_2^2 \tag{4.30}$$

This is the input-output mapping for the fuzzy controlled system. The FLC in the closed-loop configuration is to ensure that this input-output mapping negative definite, for all regions of the state space, in order to reach a global stability conclusion, if this is what is sought;

otherwise, only local stability conclusions may be reached. From (4.30), it is clear that the stability issue reduces to the regulation of the first term:

$$\left(-212.3612y_1 + 166.2865y_2\right)u_{FLC} \leq 0 \tag{4.31}$$

by the fuzzy logic controller. If this regulation holds for all y_1 and y_2, then the fuzzy logic controller is globally asymptotically stable. As was done in the nonlinear example, for any initial conditions in the state space, the FLC was expected to produce control inputs that will lead to an asymptotic stability of the closed-loop system. The regions of stability, based on the FLC's regulation of expression (4.31), were analyzed and shown in Figure 4.15. For all initial conditions on the line $M: y_2 - 1.277y_1 = 0$, the fuzzy logic control system is therefore expected to be asymptotically stable, regardless of the fuzzy controller's prescribed inputs. This line is, therefore, the *primary invariant manifold* [49]. Analytically, this is seen to be obvious, since on this line, $dV/dt = -1/2(x^T Q\, x)$, which goes to zero; where Q already solved the Lyapunov's equation for the stable, unforced system. Thus, any control profile prescribed by the FLC while the states are on this line should not contradict the known analytical result. To see this, points were taken on this line in the first quadrant (I) and the third quadrant (III) and the system was simulated with the FLC in the closed-loop. The results are shown in Figure 4.16(a) and Figure 4.16(b), and Figure 4.17. Finally, a couple of sample points were taken: 1) r = 0.15, β = 0.1, to illustrate the situation in Figure 4.15(a), quadrant I, and 2) r = 0.3, β = -0.15, to illustrate the case of Figure 4.15 (b), quadrant IV. The results are shown in Figure 4.18 and Figure 4.19.

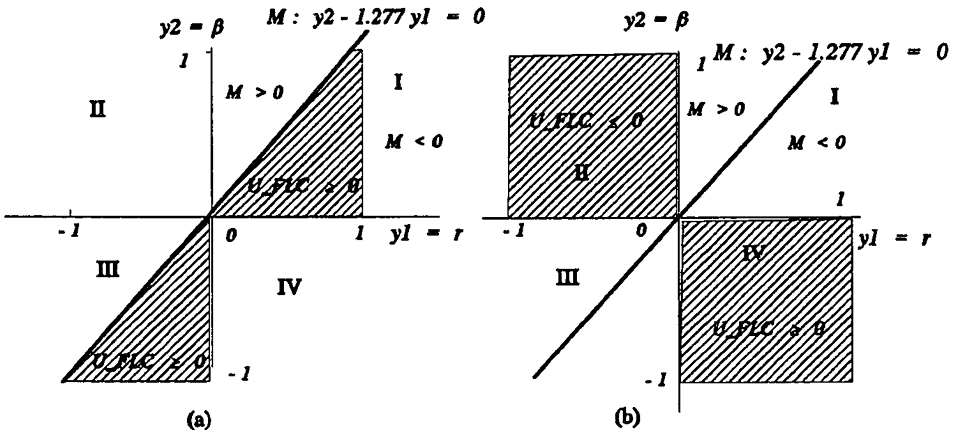


Figure 4.15(a) The FLC's Stability Regions : y_1, y_2 both $>$ 0 or bothe $<$ 0. In the shaded regions, the FLC should produce controls that are $\geq$ 0. The supplementary region in the one above M = 0. Here, the expected controls should be $\leq$ 0. (b) Definite sign for M: Occurs when y_1 and y_2 have opposite signs. With (y_1, y_2) in quadrant II, the controls should be $\leq$ 0. In quadrant IV, the controls should be $\geq$ 0.

4.8.2.1 Simulation Studies

The simulation results are shown in Figures 4.16 through 4.19. These are only representative in that they have been produced to illustrate the theory. The actual stability analysis is detailed in Figure 4.15. Figure 4.16(a) shows the result of having points on the primary invariant manifold, M, as determined from the stability analysis. Even though a control profile was generated by the FLC for points on this line, the trajectories for r and β are seen to be asymptotically stable. This fact was further supported as depicted by the first rule in Figure 4.14; this is therefore an invariant rule. The same situation is depicted in Figure 4.16(b) where, in addition, dV/dt is seen to be negative definite. Note that theoretically, $dV/dt = -0.5\, x^T Q\, x$, for points on M. However, in this figure, the effect of the transients have been removed to show that dV/dt is predominantly negative definite. Figures 4.17 through 4.19 were produced to verify the stability requirement in Figure 4.15, on the control profile generated by the FLC. For the particular points chosen in the simulations, quadrants I, III and IV in Figure 4.15 suggests that the generated fuzzy control, $u_{FLC} \geq 0$. This is seen to be the case where in the simulation results as the generated controls become steady.

4.8.2.2 Simulation Results

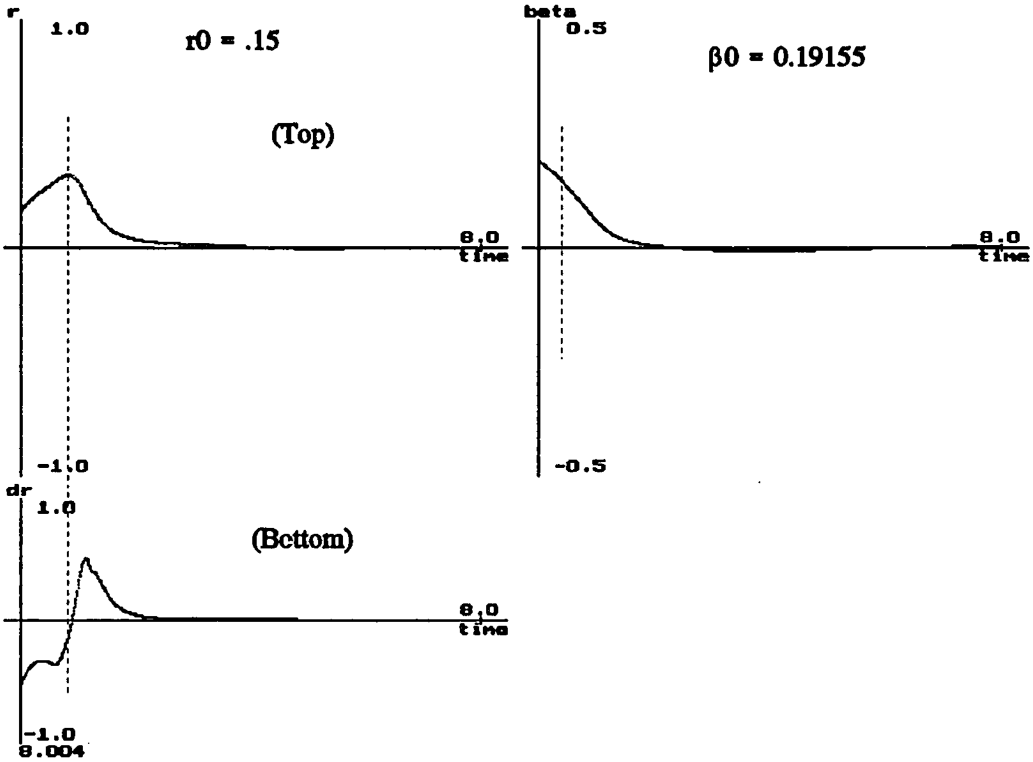


Figure 4.16 (a) Top: Trajectories for r and β with initial conditions on M. Bottom : Control profile. It has no effect on asymptotic stability for initial conditions on M. Vertical lines (dotted) are only illustrating limits of transients.

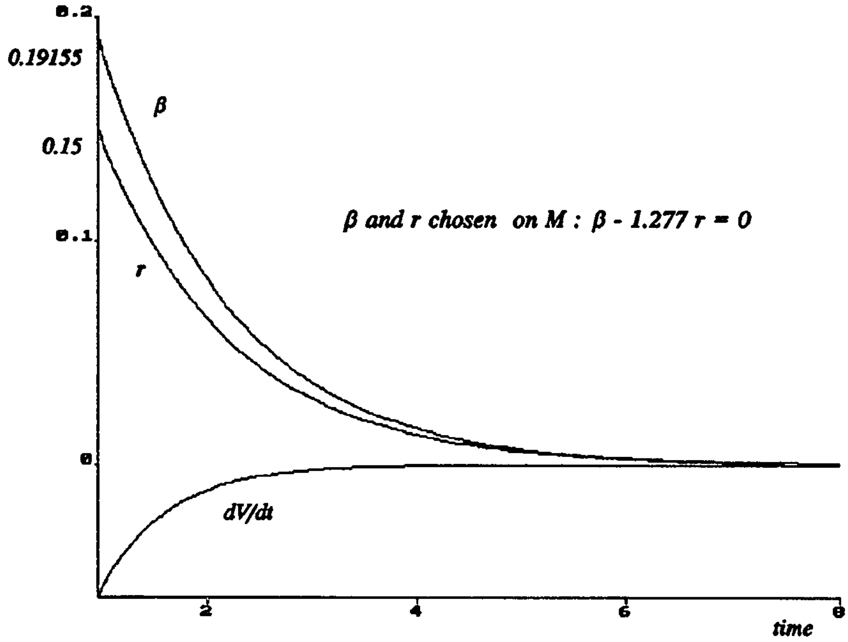


Figure 4.16 (b) Plots of β , r and dV/dt for Initial Conditions on M.

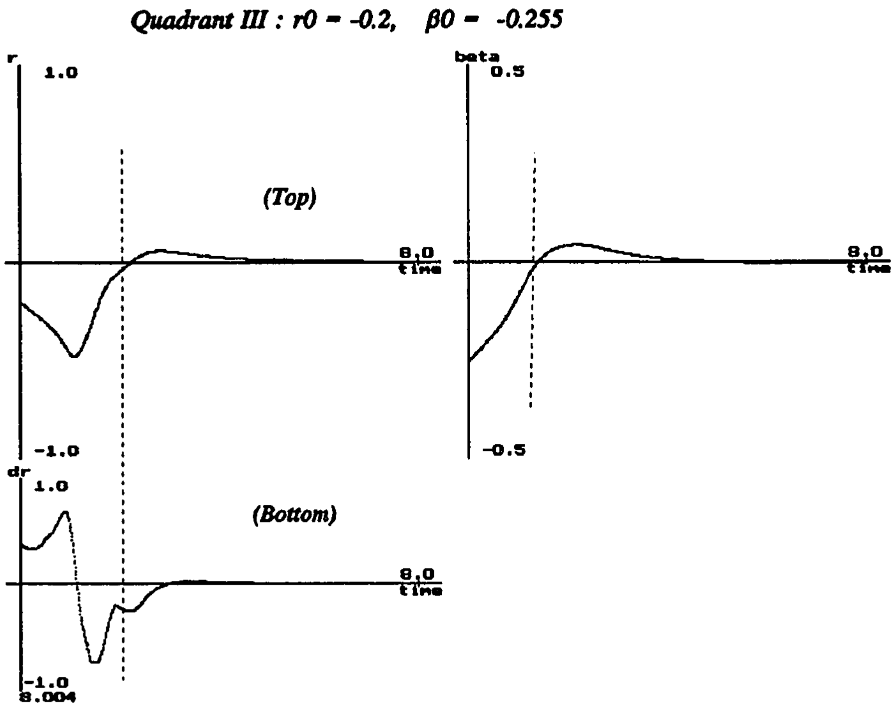


Figure 4.17 Trajectories (Top) and Controls (Bottom) in Quadrant III in Figure 4.15 (a)

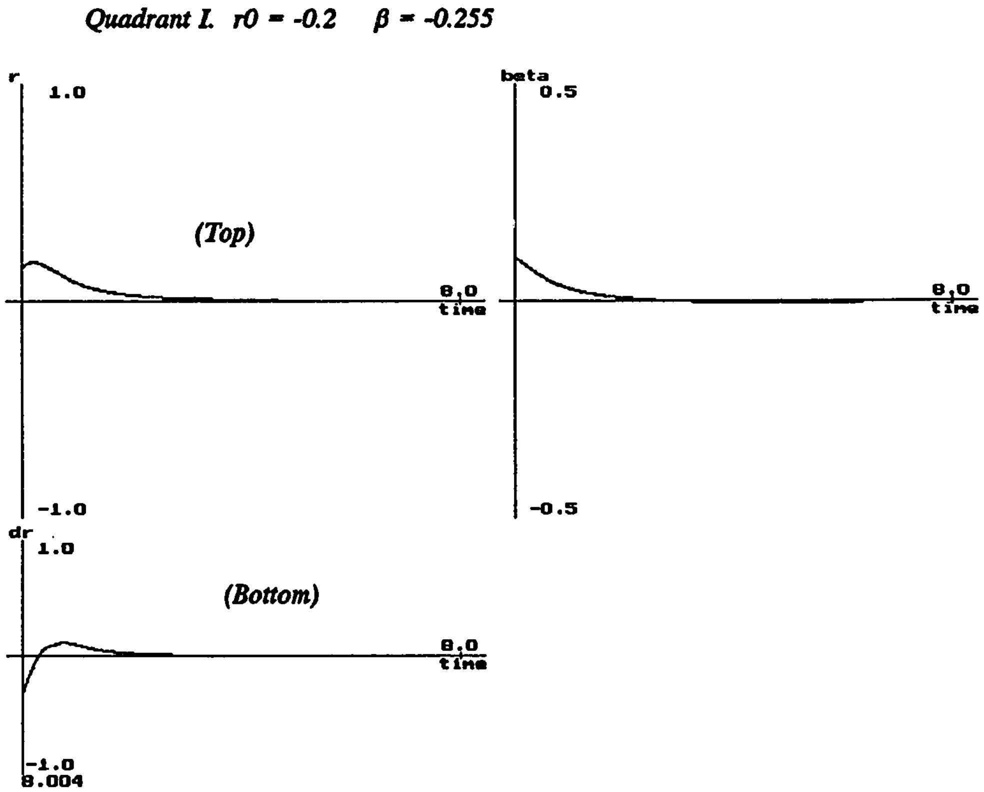


Figure 4.18 Trajectories (Top) and Control Profile (Bottom) in Quadrant III (of Figure 4.15 (a) for r anf β both positive.)

Quadrant IV : r = 0.3 , β = .1 (refer to Figure 4.15 (b)

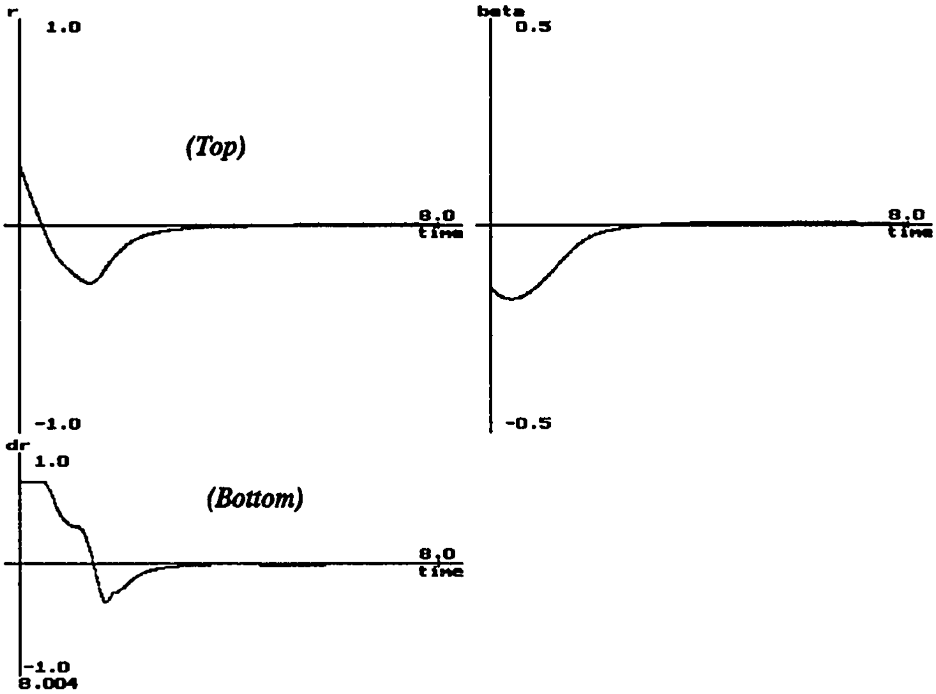


Figure 4.19 Trajectories (Top) and Control Profile (Bottom): Definite sign for M.

4.9 Conclusions

The design of the fuzzy controller for controlling the idle speed of a simplified, nonlinear two-state automobile engine model was again summarized. Moreover, the main issue of this section is the proposed stability analysis of this controller. Because of the complexity of the explicit forms of the Lyapunov function and its time derivative, one of the stability conditions in lemma 4.2 was alternatively satisfied via simulation instead of an analytical means. The results show good error convergence to the origin. Although the time derivative of the Lyapunov function shows a small initial transient in some quadrants, the overall behavior leads us to conclude negative definiteness. This, coupled with the choice of a coercive Lyapunov function show that the FLC system is asymptotically stable in the invariant set. The conclusion of negative definiteness based on a predominant behavior, which excludes the very short-duration transient, during which the time derivative is negative semi-definite, is not alarming. This would otherwise be manifested in an analytical approach of the same model as an approximation of some sort.

For the linear system example, a missile autopilot's yaw axis linearized dynamics was considered. Instead of invoking the Kalman-Yakubovich lemma directly by solving the simultaneous equations, $A^TP + PA = -Q$, $PB = C$, an input-output mapping, dV/dt, was first formulated which utilizes P and Q derived upon solving only the Lyapunov's equation. This expression was then analyzed to determine the actions that needed to be taken by the FLC, for various points in the state space. The essence of the theorem is the formulation of an input-output mapping. This was readily determined for the missile autopilot system which was stable in open-loop. The theory was used to determine the stability regions of the system. The results were seen to be in accord with the analytical results.

4.10 Summary

The methods of stability analysis developed in this chapter pertained to the successful formulation of an input-output mapping from an energy-like expression for the driven system. A theorem was stated for guaranteeing that this energy-like function is negative definite. For nonlinear systems, there is no unique way to determine this mapping. However, by identifying the outputs of the system from the available states, these are explicitly incorporated into the energy-like expression. Since the system is driven by the fuzzy logic controller in closed-loop, this expression must contain the fuzzy control variable. The task of ensuring that this energy is diminished only at the specified origin, is assumed by the fuzzy controller. It was stated that for relatively simple systems, the analytical proof of one of the requirements of the theory, namely the negative definiteness of dV/dt, may be easy to show. However, for complex dynamical system, the undue approximations that would usually be employed in the proof process may be either infeasible or physically unrealistic. Thus, the expression may have to be simulated for various points in the state space. Once this is done, the remaining requirements may be easily satisfied by an analytical means. Nevertheless, simulation results should reconfirm this fact. For stable linear system, the positive real lemma (Kalman-Yakubovich) may be used to prove dissipativeness. The only difficulty is that simultaneous equations need to be solved involving the Lyapunov's equation and the second condition on P, B and C. If the system is stable, one can always solve this, but it is very difficult to have the specified output matrix, C, automatically producing a P that solves the Lyapunov's equation. If this is the case, one may have to accept the resulting input matrix, which may be different from the specified one. This can always be rescaled to recover the signals required as the output. Equivalently, it may be easier to formulate an input-output mapping for the controlled system, and the control inputs generated by the FLC should ensure that this is at least negative semi-definite.

CHAPTER V

STABILITY ANALYSIS OF NEARLY LINEAR FUZZY CONTROLLED PROCESSES

5.1 Introduction

In this chapter, an input-output stability analysis is developed from a different perspective for a nearly linear plant. It should be emphasized that in all of the stability analyses, the implicit equilibrium states are studied, instead of the zero equilibrium state of an unforced system, in the classical sense. The system being analyzed is actually being driven by fuzzily derived controlled inputs. The stability of such a system will therefore be referred to as the stability of a fuzzy controlled system. It is assumed that the nonlinear process model is available either for analysis or for design. The nearly-linear assumption is verified for a class of physical process set-point control processes. Once this is verified, it remains to show that the system is dissipative from an input-output point of view. The stability requirements are verified analytically, where possible, or via simulation on the process model. The theorem extends to a rulebase designed either by the operator manual type if-then rules or other more systematic design approaches, if an approximate model of the process is available. It should be noted that it is not a given that a control rule which is considered to be stable in the fuzzy domain, where the control law is either linguistic in nature, or is given in some numerical or functional form, should automatically satisfy an analytical stability criterion.

Before a concrete claim can be made, these control rules should be tested for stability where an approximate model is available.

5.2 Process Set-Point Control Setting

A block diagram of a process set-point control system is shown in Figure 5.1.

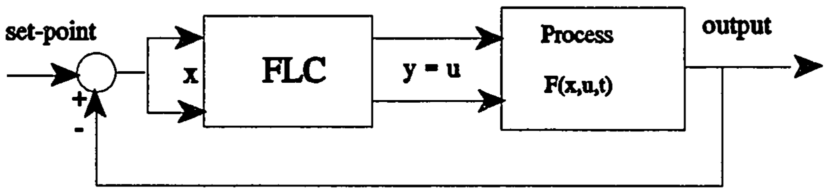


Figure 5.1 Fuzzy Logic Process Control System

The form of the rules for 2 monitored variables and two control inputs is given as:

Rule i: If x_1 is A_{i1} ... x_n is A_{in} then y_1 is B_{i1} ... y_n is B_{im}

The controller inputs $x_1,\ldots,x_n$ could be chosen to be the error, e, the change in error, Δe, integral of error, etc. The fuzzy logic controller (FLC) outputs, $y_1,\ldots,y_m$ are the control inputs to the process, F. A_{i1} ,..., A_{in} are the linguistic (fuzzy) sets or membership functions for $x_1,\ldots,x_n$ respectively, while $B_{i1},\ldots,B_{im}$ are those for the controller outputs. Let us denote, as before, F(X) and F(U) as the set of feasible states and feasible inputs respectively. By looking at the overlay of the control rules on the phase plane, the "control laws" are generally represented as a sequence of linguistic (or numerical or functional) control "values" needed for proper process operation. Typically, these could take the form given below.

$$u_n(k) = \{ NL, NM, NZ, PZ, PM, PL, \ldots \}$$

where NL = negative large, NM = negative medium, NZ = negative near zero, PZ = positive near zero, PM = positive medium, PL = positive large,etc. This is explained in some detail in Chapter IV. The sequence of "values" for the individual control input need not be identical; the above is only an illustration. Also, note that these sequential values are equally obtainable in crisp, numerical form via defuzzification. Basically, what the sequences are saying is that $\forall\, x \in F(X)$, there exists a control input sequence to be applied such that the observed process behavior is desirable. In other words, the system is controllable and accessible. Linguistically, it might be that the application of the control sequences either by a manual operator or otherwise, leads to predictable or desirable trends in the process variables $x_1,\ldots,x_n$, from which the entire process may be concluded as being proceeding "smoothly". In the neighborhood of the process desired goal, or equilibrium state, usually very little control activity may be required. This being the case, it is essential that the process be particularly stable in this near hands-off vicinity. Elsewhere in the state space, the process may exhibit a somewhat different behavior. For example, a greater control activity may be required to monitor, in particular, far-off initial states, which could lead to some nonlinear behavior because of the coarse or rough control action. This is not an uncommon procedure in process set-point adjustment at the outset. This may be summarized by saying that in the vicinity of the equilibrium state, asymptotic convergence to the set-point, is of prime concern. Elsewhere in the state space, one may be content to have bounded nonlinearity and error such that the ratio of the former and the latter is vanishingly small for all fuzzy control inputs. This will be elaborated upon in the proceeding.

Under this setting, the following is considered. The class of systems considered is the same as already described in Chapter IV. For simplicity, consider the approximate model as:

$$\dot{x} = F(x(t), u(t)), \; x \in \mathbb{R}^n, \; u \in \mathbb{R}^m, \; t \in \mathbb{R}_+; \; F: \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n \tag{5.1}$$

Let the desired set-point be x_d. Define the error by $e = x(t) - x_d$. Based on the preceding discussion, it is appropriate to assume that in the vicinity of the desired goal state, uniform control input sequences predominate for both $u_1, \ldots, u_2$, say,

$$u_{1n} = \{NZ, NZ, NZ, \ldots, NZ\}, \ldots, u_{2n} = \{PZ, PZ, PZ, \ldots, PZ\}, \; \forall k.$$

5.3 Nearly-Linear Decomposition

Let the equilibrium set of (5.1) written in implicit form below, be composed of $x = x_d$ for some nominal $u = u_e$ determined according to:

$$\{F(X,U) = 0, \Rightarrow X = x - x_d = 0, \; U = u - u_e = 0\}. \tag{5.2}$$

Denote the equilibrium set as:

$$\Omega = \{(x_d, u_e)\} \tag{5.3}$$

Let F in (5.1) be continuous in x, and its first partial exists and be continuous in x and u. Furthermore, let these be defined at all u $\in$ F(U) $\subset \mathbb{R}$. From (5.2),

$$F(X,U)|_{\Omega} \triangleq F(x_d,u_e) = 0, \ \forall t. \tag{5.4}$$

By applying Taylor's series expansion of F about Ω, we have:

$$F(x,u) = F(x,u)|_{\Omega} + \frac{\partial F(x,u)}{\partial x}|_{\Omega}(x-x_d) + \frac{\partial F(x,u)}{\partial u}|_{\Omega}(u-u_e) + g(x,u,t_0) \tag{5.5}$$

Define:

$$\frac{\partial F(x,u)}{\partial x}|_{\Omega} = A \in \mathbb{R}^{n\times n}, \ and \ \frac{\partial F(x,u)}{\partial u}|_{\Omega} = B \in R^{n\times m}, \tag{5.6}$$

$$g(.) \in \mathbb{R}^n \ is \ the \ nonlinear \ residual.$$

So (5.5) becomes:

$$\Delta\dot{x} = \frac{\partial F(x,u)}{\partial x}|_{\Omega}\Delta x + \frac{\partial F(x,u)}{\partial u}|_{\Omega}\Delta u + g(x,u), \ or \tag{5.7}$$

$$\Delta\dot{x}(t) = A\Delta x(t) + B\Delta u + g(x,u),$$

with $x - x_d$ defined as the error, e. For the purpose of illustration on a two-dimensional space, consider $\mathbb{R}^2$. The 2-D decomposition is illustrated in Figure 5.2.

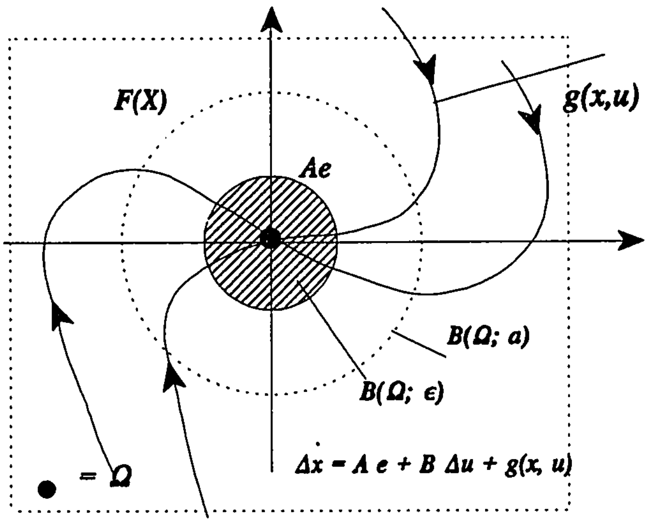


Figure 5.2 A Nearly-linear Decomposition

The residual g can be constructed, in a manner to preserve (5.1), as:

$$g(x,u,t) = F(x,u,t) - A\,e(t) \tag{5.8}$$

Comment: This comment is to help clarify the proposed development. It is appropriate to visualize that as the system evolves for any x, the control input is continually generated as a sequence of real numbers, obtained via defuzzification, and "tagged" along like a "parameter", for all incoming crisp variables, x. This parameter sequence is required to be bounded in the feasible control space, F(U), and the process is therefore required to be well defined on F(U). On the other hand, since the entire behavior of the system is of concern in the neighborhood of the origin, $\partial F/\partial x$ is required to exist and be defined at some set $\{(x; u)\} \in F(X) \times F(U)$. The set of interest is the equilibrium set $\Omega = \{(x_d; u_e)\}$.

5.4 The Stability Theorem

Suppose conditions (i), (ii) and (iii) are satisfied with $q = \Delta u B + g$ where g is given in (5.8):

i) $g(x,u)$ is continuous for $\|e\|_2 \leq a$, $t \in [0, \infty)$.

ii) $\lim \|g(x,u)\|_2 / \|e\|_2 = 0$ as $\|e\|_2 \to 0$ uniformly with respect to u.

iii) and if the system in (5.7) is dissipative

then the solution, $e \equiv 0$, of the system (5.1) is asymptotically stable. If a is the whole of F(X), then global asymptotic stability results. Condition (iii) is verified using the method of Chapter IV, Section 4.7.1, for verifying dissipativeness. As is shown in Chapter IV, the input-output mapping:

$$\frac{dV}{dt} = e^T PBu - \frac{1}{2} e^T Qe, \quad A^T P + PA = -Q. \tag{5.9}$$

needs to be shown to be negative definite, for $u = u_c$, $x = x_d$. One may only be able to show conditions (i) and (ii) analytically if the expression for the residual, g, is fairly uncomplicated. Thus, for the particular system, it is always easy to simulate these functions and draw conclusions from the results. A general proof of the theorem is given next.

Proof

Let rewrite (5.7) for convenience as:

$$\dot{e}(t) = Ae(t) + q(e,u), \quad q = B\Delta u + g(e,u), \quad e = x(t) - x_d, \quad u = u_{FLC}. \tag{5.10}$$

Let us shift the origin to $x = x_d$ so that $x = 0$ is the equilibrium state. One needs to show that the solution $x(t) = (t, t_0 = 0, x_0)$ is defined on $[0, \infty)$ when x is near x_d, that is, $x(t) \in F(X)$. Let $\Phi(t)$ be the state transition matrix of $\dot{x}(t) = Ax(t)$. Then by (iii), $\exists R, \alpha \ni \|\Phi(t)\| \leq R e^{-\alpha t}$, for all $t \geq 0$. Since $A \in \mathbf{R}^{n \times n}$, a constant matrix, integrating (5.10) in the new coordinates, with $x = 0$ as the origin gives:

$$x(t) = \Phi(t)x_0 + \int_0^t \Phi(t-\tau)q(x(\tau),u(\tau))d\tau \tag{5.11}$$

$$\begin{aligned} \|x(t)\| &\leq \|\Phi(t)x_0\| + \left\|\int_0^t \Phi(t-\tau)q(x(\tau),u(\tau))d\tau\right\| \\ &\leq Re^{-\alpha t}\|x_0\| + \int_0^t Re^{-\alpha(t-\tau)}\|q(x(\tau),u(\tau))\|d\tau, \quad or \qquad (5.12) \\ \|x(t)e^{\alpha t}\| &\leq R\|x_0\| + \int_0^t Re^{\alpha\tau}\|q(x(\tau),u(\tau))\|d\tau \end{aligned}$$

This expression holds for $t \in [0, T)$ for which $\|x(t)\| < a$ in (i). From the stability condition (iii), given any $l > 0$, $\exists\, r > 0, \ni$ for $t \geq 0$ and $\|x(t)\| < r$, the residual, $\|q(x,u)\| \leq l\ \|x(t)\|$. This means that $\|B\Delta u + g(x, u)\|$ is bounded. The quantity $B\Delta u = \partial F/\partial u|_0$ is known to be bounded, so one needs to worry about $\|g(x,u)\|$. Suppose the initial condition is such that $\|x(t)\| < r$ for $t \in [0, t_1)$, then the last inequality in (5.12) becomes:

$$\|x(t)\|e^{\alpha t} \leq R\|x_0\| + \int_0^t lRe^{\alpha\tau}\|x(\tau)\|d\tau, \quad t \in [0,t_1]. \tag{5.13}$$

This expression is in a form that can be manipulated using the Gronwall's Lemma for establishing the boundedness of signals which is stated below.

Lemma 5.1: (Gronwall, [91])

Suppose y(t) is a continuous real valued function that satisfies $y(t) \geq 0$ and:

$$y(t) \leq b + a \int_0^t y(\tau)d\tau, \quad t \in [0, t_1], \tag{5.14}$$

where a, b are positive constants. It then follows that $\forall\, t \in [0, t_1)$,

$$y(t) \leq be^{at}. \tag{5.15}$$

in expression (5.13), denote $y(t) \triangleq \|x(t)\| e^{\alpha t}$, $b \triangleq R\|x_0\|$, and $a \triangleq lR$. Clearly, a and d are positive, and $\|x(t)\| e^{\alpha t}$ is real valued and continuous. It follows that:

$$\begin{aligned} \|x(t)\| e^{\alpha t} &\leq R\|x_0\| e^{lRt}, \quad 0 \leq t < t_1, \quad or \\ \|x(t)\| &\leq R\|x_0\| e^{(lR-\alpha)t}. \end{aligned} \tag{5.16}$$

Thus $x(t)$ is bounded above. Since x_0, l are ours to choose, let us choose l such that the exponent in (5.16) is negative, that is, $(lR - \alpha) < 0$, or $lR < 0$, and $x(0) = x_0$, so that $\|x_0\| < (r/2R) \Rightarrow \|x\| < (r/2)$, $0 \leq t < t_1$. Since $q(x,u) = B\,\Delta u + g(x, u)$ is defined for $\|x\| < a$ and $0 \leq t < \infty$, this implies that the solution x(t) which exists locally at every point (t, x), t > 0, $\|x\| < a$, can be extended interval by interval so as to preserve above bound. As a result, given any solution $x(t) = x(t;\, t_0 = 0, x_0)$ with $\|x_0\| < (r/2R)$, it is defined on $0 \leq t < \infty$, and satisfies $\|x(t;\, t_0 = 0, x_0)\| < (r/2)$.

Note that one can make r as small as desired, which implies $x(t) \equiv 0$ is stable, and IR < α implies that it is asymptotically stable ■

5.5 Example

Consider the test-bed fuzzy idle speed controller for the two-state engine model in (5.10).

$$\begin{aligned} \dot{x}_1 &= k_p[a_0x_2+a_1x_1-a_2x_1x_2-a_3x_2x_1^2+g(x_1)a_4] + [k_pg(x_1)a_5]u_1 \\ &\quad + k_p[g(x_1)a_6u_1^2] \triangleq F_1(x,u,T_d) \\ \dot{x}_2 &= k_N[-a_7+a_8\frac{\dot{m}_{ao}}{x_2}+a_{12}x_2-(a_{13}+a_{14})x_2^2] + [a_{10}x_2+a_{11}]k_Nu_2 \\ &\quad +[-a_9k_N]u_2^2+a_{15}T_d \triangleq F_2(x_2,u,T_d) \end{aligned} \tag{5.17}$$

where

$$x=(x_1\ \ x_2)^T \triangleq (P\ \ N)^T\ ,\qquad u=(u_1\ \ u_2)^T \triangleq (\theta\ \ \delta)^T \tag{5.18}$$

The errors in P and N are defined as:

$$e_p = P-P_0 = x_1-x_{1d},\quad and\ e_N=N-N_0=x_2-x_{2d} \tag{5.19}$$

The general form of the linguistic rules for the controller in [61] was given as:

If (N is F_1) & (P is F_2) Then ($\Delta\delta$ is G_1) & (Δ is G_2).

where F_i and G_i are the consequent and premise linguistic terms respectively.

For this system, the equilibrium state is $x = x_d$ = (34.250 kPa, 750 rpm). For this rule base, the fuzzy control is uniform in the region around the equilibrium point. The sequence of fuzzy control in this vicinity is u_{1n} = {6.01} and u_{2n} = {28.1}, for all instant time, k. The linearized system is given by:

$$\Delta\dot{x} = \begin{bmatrix} -15.147 & -0.8377 \\ 69.68 & 1.3059 \end{bmatrix} \Delta x + \begin{bmatrix} 89.379 & 0 \\ 0 & 9.366 \end{bmatrix} \Delta u, \qquad (5.20)$$

$$\Omega = \{(34.250, 750)\}.$$

To study the stability of the system, under the applied action of the fuzzy rule base, it suffices to show that:

- $A\,|_{xd,\,ue}$ is stable for $\mathbf{u}_e \in$ { (NZ,NZ,...,NZ), (PZ,PZ,...,PZ) }
- lim $\|g(x,u,t)\|_2 / \|e\|_2 = 0$ as $\|e\|_2 \to 0$, for $(\mathbf{u},x) \in \{F(U)\backslash u_e\} \times \{F(X)\backslash x_d\}$.

5.5.1 Simulation Studies and Proof of Stability

Let us start with condition (iii). The matrix A has been found to be:

$$A = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} \end{bmatrix}_{\{(34.25,750),(6.01,28.1)\}} = \begin{bmatrix} -15.147 & -0.8377 \\ 69.9253 & 1.3059 \end{bmatrix} \quad (5.21)$$

This Jacobian matrix is nonsingular; the eigenvalues are -9.9368 and -3.9044. Thus A is stable. Hence $\exists\, P,Q > 0 \ni A^TP + PA = -Q$. Solving this equation with $Q = I_2$ gives:

$$P = \begin{bmatrix} 2.8548 & 0.61107 \\ 0.61107 & 0.031 \end{bmatrix}, \quad with\ Q = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (5.22)$$

If no restriction is placed on the choice of the output matrix C, it may be computed directly from the *positive real* condition $C = B^TP$. This is computed as:

$$C = B^TP = \begin{bmatrix} 255.1592 & 54.616 \\ 5.7233 & 0.2903 \end{bmatrix} \quad (5.23)$$

The output is then $y = Cx$. From this, it seems as though one has no choice in the selection of states to be used as outputs. One way to circumvent this is to device an auxiliary output, z, and then scale the above output accordingly. Therefore, an input-ouput mapping exists and it is dissipative. This concludes condition (iii).

The function g in this case is constructed as:

$$\begin{aligned} g_1 &= F_1(x,u,t) - (\frac{\partial F_1}{\partial x_1} e_p + \frac{\partial F_1}{\partial x_2} e_N) \\ g_2 &= F_2(x,u,t) - (\frac{\partial F_2}{\partial x_1} e_p + \frac{\partial F_2}{\partial x_2} e_N) \end{aligned} \tag{5.24}$$

Condition (i): F is continuous in x and u as a consequence of the existence of the jacobian matrices, A and B which are continuous at Ω and conversely. By a theorem for algebraic combination of continuous functions, it follows that both g_1, and g_2 are each continuous in the same set ■

The Euclidean norm is used on g and e, that is:

$$\begin{aligned} \| g(.,.) \|_2 &= (g_1^2 + g_2^2)^{1/2} \quad and \\ \| e(.) \|_2 &= (e_P^2 + e_N^2)^{1/2} \end{aligned} \tag{5.25}$$

Finally, condition (ii) is satisfied for various initial conditions, in the state space, with their associated fuzzy control sequences, via simulation of the norm ratios. For relatively simple systems, it may not be too difficult to evaluate, analytically, the limit of $\| g(.,) \| \ / \ \| e(.) \|$, as $\| e(.) \| \to 0$. However, for this system, a lengthy and possibly "brute force" algebra is avoided by computer simulation of the ratio. Where the norm of the error does go to zero, the ratio is checked to see if, it too, goes to zero, thus satisfying condition (ii). One result of this procedure is provided as an example. The initial point, (x_1, x_2) = (51.0 kPa, 1050 rpm), falls in a region with the fuzzy control sequence (u_1, u_2) = (26.1, 4.01) degrees. These values are substituted into the norm ratio and simulated. The errors and the ratio are seen to both converge to zero, as shown on Figure 5.3. It is concluded from this that the fuzzy control values drive the system exponentially to a stable equilibrium at the origin, and that the control rule rules elsewhere, in the state space, lead to bounded and diminishing nonlinearity. The significance of satisfying (ii) is that one is now able to say that $\| g(.,.) \| = O. \|e\|$, uniformly with respect to u, as $\|e\| \to 0$. Results of this simulation show that $O = 4.61823 \times 10^{-5}$, which is indeed of order zero. This is the most crucial of the two conditions to satisfy, because it implies that **F**(t, 0) = 0, hence $e(t) = 0 \Rightarrow x_d = (34.24, 750)$, for the subset of fuzzy rules considered, is asymptotically stable. Satisfying this for all $u \in F(U) \setminus u_e$ and $x_0 \in F(X) \setminus x_d$, implies stability of the rule base. The fact that F is differentiable at x_d ensures the uniqueness of this solution, which is the equilibrium point.

5.5.2 Simulation Results

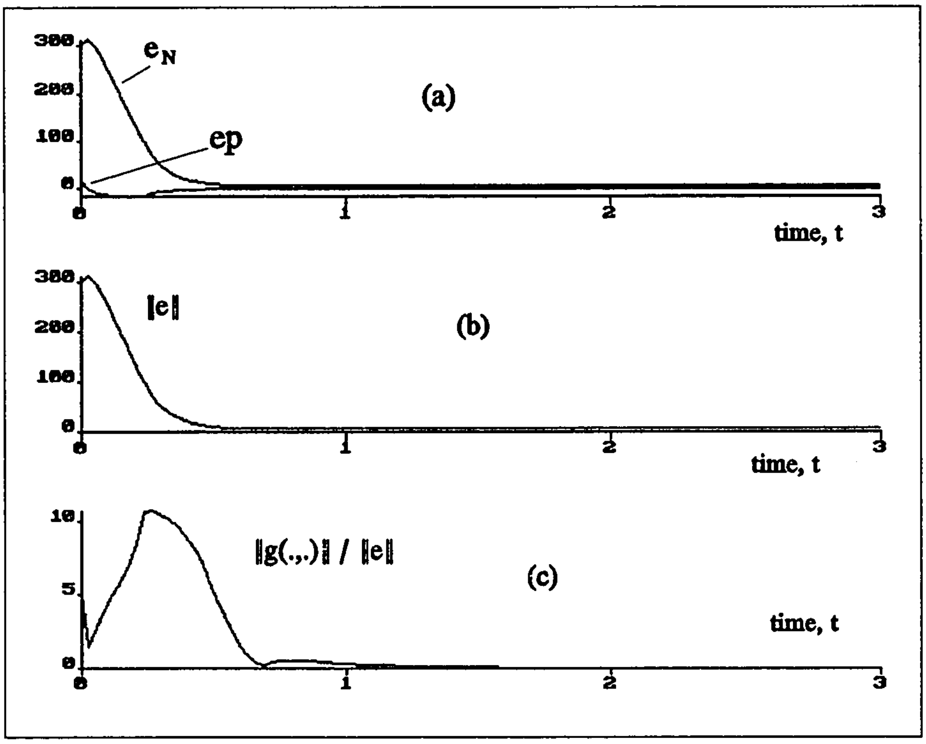


Figure 5.3 (a)Errors in N and P (b)Norm of e (c)Norm Ratio

5.6 Summary

The developed theorem employed here is a statement of L_2 stability for the feedback system, where the control inputs are fuzzily derived. An important fact to remember when dealing with fuzzy control systems is the implicit system should be studied for stability. This system is, in addition, being driven by controls that are fuzzily derived and are continually being generated as the system evolves. The structure of the "control law" for the class of fuzzy controllers considered in this chapter is such that several regions of the state space have various linguistic control rules. These control rules will have to be tested exhaustively in the corresponding regions of the state space. The analysis becomes less exhaustive if the fuzzy control is uniform over a large region. This perhaps should be a key difference between fuzzy control system stability and the conventional control system stability analyses. The example considered on the test-bed fuzzy controller shows how the theory may be applied to study the stability of the fuzzy control system. If the system is linear, it suffices to only show that an input-output map can be formed which is dissipative as discussed in Chapter IV.

CHAPTER VI

A ROBUSTNESS ANALYSIS OF FUZZY LOGIC CONTROL SYSTEMS

6.1 Introduction

An approximate method is formulated for analyzing the performance of a broad class of linear and nonlinear systems controlled using fuzzy logic. It is assumed that an approximate model of the nominal plant is available. Stability in the presence of small, bounded parametric uncertainty and external disturbance of a known origin is considered. The method is based on an appropriate formulation of an expression for the time derivative of a Lyapunov-like function of the nominal plant in terms of the error and a heuristic measure of the system's error sensitivity with respect to the parameters of interest. This is be considered as the performance measure which is to be minimized. By applying a Lyapunov stability criterion, the robustness of the system is analyzed by observing a definiteness condition of a simple matrix. The theory is developed in two parts. First, it is assumed that the states are decoupled in the sense that a parameter perturbation in a particular state has no effect on other states. The main robust stability result is developed based on this assumption. In the second part, a weak interaction is allowed to exist between states as a result of parameter variation. Estimates of the measure of the interactions are derived on stability grounds. A more general result for robust stability is then given. The point of stability convergence is the system's specified target point. Thus, stability convergence to this point, in the presence of parameter perturbations and external load inherently addresses robust performance of the closed-loop system. Finally, inequality bounds are derived for the sensitivity, error deviation and parameter deviations in terms of fuzzy quantities. A measure for robustness is then formulated using singular values.

6.2 Remarks

One astonishing thing about the work in fuzzy control is that most researchers have not been relentlessly capitalizing *directly* on one of the most mentioned reasons for using fuzzy logic controllers, which is: lack of an accurate model, but the *availability* of only an approximate dynamical model of the plant. It would seem reasonable and convincing to use a fuzzy logic controller along with an approximate, if only fairly accurate dynamical model of the plant, but with precise performance specifications, and see how the entire closed loop fuzzy system fares with a nonfuzzy controller using a more or less accurate dynamical model of the plant. An attempt is made to bridge this gap in the proceeding.

Because the formulation is based on the time derivative of the system's Lyapunov function, we are explicitly alluding to an intuitive physical notion of stability. That is, the total energy of a dynamical system decreases monotonically until a state of equilibrium is reached. As reported in [54], the energy of a fuzzy set depends on its support and shape of its membership function. Thus, it is important that any discussion of stability and therefore of robustness of a fuzzy system incorporate these elements, whether the analysis is carried out entirely in a fuzzy domain or in a hybrid domain. By hybrid, it is meant both fuzzy and nonfuzzy (deterministic), to distinguish this from another usage which considers hybrid to be fuzzy and stochastic. The essence of fuzziness of the system needs to be captured and incorporated in the formulation, possibly via a direct utilization of membership functions of the pertinent systems variables. However, if the membership functions are too complicated to be handled directly in the analysis, the resulting crisp output of the closed loop system should suffice for analysis. This is true as long as a fuzzy controller is used to close the loop. This controller should also be the *defacto* stabilizer of the system under the various operating conditions. Such an approach is not only relevant and practical, but also establishes a baseline for comparison with other types of controllers that could be used to stabilize the system.

6.3 Requirement for Robustness

For robustness analysis, the following will be required: (i) A controller, K. (ii) A set of plants F, and (iii) a desired property or characteristics, Θ. The robustness set is therefore the set of triplet denoted by:

$$\mathfrak{R} = \{K, F, \Theta\} \tag{6.1}$$

Any of these elements can be generic in nature. For instance, in this case, K is a fuzzy logic controller. In other words, it is a rule base, a set of fuzzy if-then rules that has been designed for a given plant $f \in F$ using the predominantly essential procedures: fuzzification, inferencing using an appropriate knowledge base and defuzzification [4],[6].

6.3.1 Robustness Problem Statement

Given a fuzzy logic controller, K, in the robustness set $\Re$, the following three things are required.

1. K to stabilize the nominal plant, $f \in F$, where all the parameters of f are fixed at their nominal values.
2. K to stabilize the perturbed plant $f + \Delta f$, where $|\Delta f|$ is bounded and is due to all reasonably and practically possible perturbations in the plant's parameters of interest, including the onset of bounded external disturbances of known origins.
3. Obtain an approximate, quantitative measure of the system's robustness, including bounds on the perturbations and the error sensitivities.

Upon knowing the bound of the measure in (3) above, its universe of discourse can be determined for the particular system. In addition, one will be able to express the robustness measure in a fuzzy sense such as very robust, robust, weakly robust, etc. This description may be viewed similar to, for instance, the relative stability measure that gain and phase margin offer in classical control theory. The bound of the measure, on the other hand, may be viewed similar to μ-analysis in modern control theory. The difference is that the formulation here, under certain stringent but reasonable assumptions, applies to a wide class of both linear and nonlinear fuzzy controlled plants, and not just linear plants.

6.4 Concepts of Sensitivity and Robustness

It is not uncommon to find sensitivity and robustness being used (or rather misused) synonymously. The two are different. On one hand, sensitivity of a system is usually discussed with respect to a certain parameter of the system. In other words, it is a statement of how the system as a whole or in part, responds to a parameter variation. It is often expressed as the ratio of the fractional change of an observed system's variable to a fractional change in the parameter that has caused the change. Therefore, in discussing sensitivity, nothing is usually said about the stability of the system. On the other hand, robustness is always associated with a system's stable operation in the presence of internal and external perturbations. The internal perturbation invariably contains the case of parameter variations mentioned above. Robustness is therefore often used as a measure of a system's performance. This section introduces some definitions and terminology that will be utilized in the development of robustness measures of a closed-loop system. The dynamical system may be represented as a 6-tuple:

$$\sum :(f,h;x,\alpha,u,d) \qquad (6.2)$$

where f is a nonlinear operator, x $\in \mathbf{R}^n$ is the observable state vector, $\alpha \in \mathbf{R}^r$ is the real parameter vector, u $\in \mathbf{R}^m$ is the input vector, d $\in \mathbf{R}$ is a scalar disturbance and h is the nonlinear output map. The system, as an input/output map, is expressed as:

$$\sum : f = \dot{x} - F(x,u,\alpha,d) = 0, \quad y = h(x,u,\alpha,d) \qquad (6.3)$$

This is illustrated in on Figure 6.1.

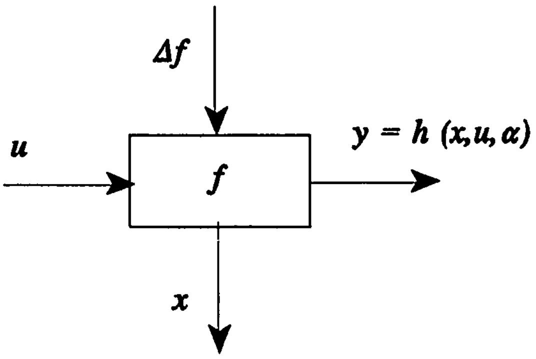


Figure 6.1 A Dynamical System Representation

where $\Delta f \triangleq \{\Delta\alpha, \Delta x, d\}$ represents the perturbation set. Furthermore, let u, α, x and d $\in$ some normed linear space, and such that $\|u\| < M_u$, $\|\alpha\| < M_\alpha$, $\|x\| < M_x$ and $\|d\| < M_d$, with M_u, M_α, M_x and M_d all $< \infty$. As before, $x \in F(x)$, the set of feasible states, $u \in F(U)$, the set of feasible inputs. The set of admissible parameters will be denoted by $F(\alpha)$. Similarly, the set of physically allowable parameter perturbation is denoted by $F(\Delta\alpha)$. Since one is concerned with the behavior of the system under the mappings f and h, for all u, and a specified α, it is only appropriate to assume that f and h can vary in a neighborhood of some α_0, the nominal parameter value.

Definition 6.1: *Insensitivity*: The system is considered insensitive with respect to $\alpha = \alpha_0 + \Delta\alpha$ perturbation if $\forall\, u_0 \in F(U)$, y depends continuously on α. If $\|y - y_0\|$ is small, $\forall u_0$, fixed, and $\alpha \neq \alpha_0$, then the system's output is insensitive with respect to $\alpha + \alpha_0$.

Figure 6.2 illustrates the setting of this definition.

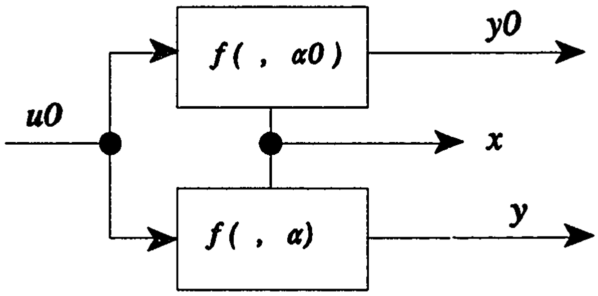


Figure 6.2 Parameter Sensitivity.

Definition 6.2: *Robust*: The system is robust to parameter variation if $\forall\alpha \in$ the ball, $B(\alpha_0, \Delta\alpha)$, the system is stable from an input/output point of view. That is, the system is stable at $\forall\alpha \in B(\alpha_0, \Delta\alpha)$, if $\|y - y_0\|$ is small for $\|\alpha - \alpha_0\|$ is small and such that (6.3) holds for fixed $u \in F(U)$.

Usually, stability of the system at α means that $\|y_1 - y_2\|$ is small whenever $\|u_1 - u_2\|$ is small, such that (6.3) is satisfied for u_1 and u_2, and α. Clearly, this pertains to some continuity of the input/output maps, f and h. These concepts will be utilized when in addition to $\Delta\alpha$, an external disturbance $d \in F(D)$ is applied. In this case the system will be considered robust if it is stable for $\alpha \in B(\alpha_0, \Delta\alpha)$ and $d \in F(D)$, and such that the error in some design specification, $e = x - x_d$, is minimum.

6.5 Outline of a Systematic Robust Analysis

- Availability of approximate process model of the form: $F(x, u, d, \alpha,)$
- Nominal Stability:
  - Design the fuzzy controller for the nominal system:

    $F(x, u_e, d_0, \alpha_0)$, $d = d_0$, α_0 = nominal parameter values of disturbance and parameter, respectively, u_e = FLC
  - Verify the stability of F under rule base u_e.
- Robust Stability:
  - Identify the Uncertainty : $F: \rightarrow F + \Delta F$
- Sources of Uncertainty:
  - Bounded Parameter variation $\alpha_0 + \Delta\alpha$
  - Unmodelled dynamics $\dot{x} + \Delta\dot{x}$
  - External disturbance, Noise

Lump all these together as $\Delta F = \{ \Delta\alpha, \Delta x$, external disturbance$\}$ as shown in Figure 6.1.

Conclusion:

If the above closed-loop system is stable for all bounded ΔF and all physically possible bounded inputs such that the desired design goal is met, then the FLC is robust.

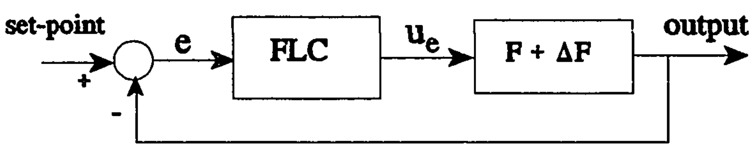


Figure 6.3 Uncertain Fuzzy Feedback System

6.6 Formulation of Fuzzy System Robustness

The task of the fuzzy control system may be broken down into stability and meeting a desired objective. This says that the fuzzy controller should guarantee a stable operation as specified objective is met. The class of systems considered is given by:

$$F: \dot{x}' = f(x', u, \alpha), \text{ where } x' \in \mathbb{R}^n,\ u \in \mathbb{R}^m,\ t \in \mathbb{R}^+,\ \alpha \in \mathbb{R}^r, \tag{6.4}$$

and $f \in \mathbb{R}^n$.

Note, the terminology and definitions developed in the previous section are general. In the following different symbols may sometimes be used; these are clarified without loss of generality.

x = system's states with their universes of discourse, $E_x \subset F(X)$.

$\alpha = [\alpha_i ; i = 1,2,\ldots,r]$ is system's parameter vector with each

$\alpha_i = [\alpha_{min}, \alpha_{max}]$.

$\alpha_{oi} \in E_\alpha \subset F(\alpha)$, the nominal value of α_i.

$\Delta\alpha_i$ = small perturbation in α_i about the nominal value.

u = input vector

Define the error as:

$$e = x' - x_d \in \mathbb{R}^n \tag{6.5}$$

where x_d is the desired state vector.

The matrix formulation used in the theory restricts that $r = n$, that is there are as many states as there are parameters, otherwise, this restriction can be removed. For simplicity and clarity, consider the case of only two parameters and two states. ie., $r = n = 2$, and define as *implicit* states the error coordinates $x_1 = e_1 = x_1' - x_{1d}$ and $x_2 = e_2 = x_2' - x_{2d}$. The generalization to $\mathbb{R}^n$ is fairly straight forward, and will be made shortly.

***Assumption 6.1*.**

Suppose that the parameter α_i is dominantly a characteristic of state i dynamics, $\dot{x}_i'$, and such that "cross-coupling" or interaction between channels is negligible. In other words, a perturbation in the parameter α has a negligible effect on the growth of error e_j, $i \neq j$. That is, e_i is not directly sensitized by the perturbation, $\Delta\alpha_j$. Therefore, one parameter can be fixed as the other is varied. This hypothesis is not difficult to satisfy on most engineered systems where some ad hoc procedures are usually employed in practical sensitivity analysis.

The result is extended later on, to the case where a very weak interaction is assumed to exist, and which is therefore considered to be vanishingly small. As a consequent of above assumption, the following definitions are stated.

Definition 6.3: Define a unit parameter perturbation of a state(or channel) as:

$$\frac{\Delta\alpha_i}{\alpha_i} = k_{ii} \in \mathbb{R}, \; i = 1,2,\ldots,r \tag{6.6}$$

Definition 6.4: The fuzzy sensitivity of the real output function, e(x, α_i,t) with respect to the real parameters α_i, i=1,...,r, is expressed by [40]:

$$S_{\alpha}^{e} = \frac{1-\mu_{\Delta e}}{1-w_1\mu_{\Delta\alpha_1}-w_2\mu_{\Delta\alpha_2}-\ldots-w_r\mu_{\Delta\alpha_r}}, \sum_{i=1}^{i=r} w_i=1, \; w_i \in [0,1]. \qquad (6.7)$$

where μ are membership functions of the deviations and the w's are weights that are heuristically attached to signify the importance of a parameter. For two parameters jointly considered and of equal importance, the above becomes:

$$S_{\alpha}^{e} = \frac{1-\mu_{\Delta e}}{1-\; w_1\mu_{\Delta\alpha_1}-\; w_2\mu_{\Delta\alpha_2}}, \quad (w_1=w_2=0.5). \qquad (6.8)$$

If the parameter α_i is attributed to state i only, then this can be viewed as the case of a single parameter per state, which can be varied independently. That is:

$$S_{\alpha_i}^{e_i} = \frac{1-\mu_{\Delta e_i}}{1-\mu_{\Delta\alpha_i}} \quad i=1,\ldots n \qquad (6.9)$$

From this and definition 6.1, the set below is formed for i = 2.

$$\{S_{\alpha_1}^{e_1}, S_{\alpha_2}^{e_2}, t;\; k_{11}, k_{22}\} \qquad (6.10)$$

Whenever cross-coupling exists, the single parameter expression will still be used but with parameter α_i giving rise to a change in e_i. In the general case the set in (6.10) assumes its most general form below:

$$\left\{S_{\alpha_i}^{e_i}, S_{\alpha_i}^{e_j}, t;\ k_{ii}\right\} \tag{6.11}$$

where $S_{\alpha 1}^{e1}$, for example, measures the sensitivity of e_1 due to α_1.

The main result is stated below as a theorem.

6.6.1 The Main Robust Stability Result

Theorem 6.1:

Let the fuzzy controller K stabilize $f(., \alpha_{oi}) \in F$, i = 1,2 (nominal stability), and suppose that assumption 6.1 holds:

Robust Performance: The fuzzy controller is robust with respect to F if the matrix formed as

$$\begin{aligned} P = [p_{ij}] = [S_{\Delta\alpha_i}^{e_j} k_{ii}],\ i=1,2,\ldots,r, \\ \text{is negative semi-definite},\ \forall t>0 \end{aligned} \tag{6.12}$$

The matrix, P, will be referred to as the per unit sensitivity matrix. For r = n= 2, this is:

$$P = \begin{bmatrix} S_{\Delta\alpha_1}{}^{e_1} k_{11} & 0 \\ 0 & S_{\Delta\alpha_2}{}^{e_2} k_{22} \end{bmatrix} \leq 0, \ \forall t > 0. \tag{6.13}$$

6.6.2 Derivation of the Main Result

Let us define the performance measure as a lower-bounded function given by:

$$\mathfrak{I} \triangleq V(e) = \frac{1}{2}(e_1^2 + e_2^2) \quad or \quad V(x_1, x_2) = \frac{1}{2}(x_1^2 + x_2^2) \tag{6.14}$$

This needs to be minimized. A global minimizer for this is $x_1 = x_2 = 0$. Suppose that $x_1 = x_2 = 0$ is the only equilibrium point of the nominal plant. From stability standpoint, the FLC is required to ensure that $\dot{V} = dV(x_1, x_2)/dt \leq 0$, for all $t > 0$, or $dV(x_1, x_2) \leq 0$. From (6.12), we have:

$$dV(x_1, x_2) = \frac{1}{2}(2x_1 dx_1 + 2x_2 dx_2) = x_1 dx_1 + x_2 dx_2 \tag{6.15}$$

But,

$$\begin{aligned} dx_1 &= \frac{\partial x_1}{\partial \alpha_1}\Delta\alpha_1 + \frac{\partial x_1}{\partial \alpha_2}\Delta\alpha_2, \quad and \\ dx_2 &= \frac{\partial x_2}{\partial \alpha_1}\Delta\alpha_1 + \frac{\partial x_2}{\partial \alpha_2}\Delta\alpha_2, \quad with \ \Delta\alpha \approx d\alpha (small\, changes) \end{aligned} \tag{6.16}$$

Consider the most general notion of sensitivity from control theory. The sensitivity of an observed variable e which is a function of the parameter α is defined as the percentage change in the quantity e divided by the percentage change in the parameter α that caused the change in e [74]. The most commonly used expression is:

$$S_{\alpha}^{e} = \frac{\partial e/e}{\partial \alpha/\alpha} = \frac{\partial e}{\partial \alpha}\frac{\alpha}{e} \tag{6.17}$$

This should give a reasonably good estimate of the sensitivity for small changes. In all cases, fuzzy or non fuzzy, the objective is to keep the sensitivity as small as possible. Ideally, a system that is completely insensitive to all parameter variation is said to have a sensitivity of zero. Comparing (6.9) and (6.17) gives:

$$S_{\alpha}^{e} = \frac{\partial e}{\partial \alpha}\frac{\alpha}{e} \approx \frac{1-\mu_{\Delta e}}{1-\mu_{\Delta\alpha}} \tag{6.18}$$

From this, it is approximated that:

$$\frac{\partial e}{\partial \alpha} \approx \left[\frac{1-\mu_{\Delta e}}{1-\mu_{\Delta\alpha}}\right]\frac{e}{\alpha} \tag{6.19}$$

This is a very important expression for it allows the partial derivative to be approximated in terms of fuzzy quantities. For notational simplicity, the deviation from the parameter α will be denoted by $\tilde{\alpha}$, instead of $\Delta\alpha$, with a membership function $\mu_{\tilde{\alpha}}$. Using (6.16) through (6.19), and using x_1 and x_2 in place of e_1 and e_2, in the expression for dV in (6.15), the expression becomes:

$$dV(x_1,x_2) = x_1\left[\frac{1-\mu_{\tilde{x}_1}}{1-\mu_{\tilde{\alpha}_1}}\frac{\Delta\alpha_1}{\alpha_1}x_1 + \frac{1-\mu_{\tilde{x}_1}}{1-\mu_{\tilde{\alpha}_2}}\frac{\Delta\alpha_2}{\alpha_2}x_1\right] + x_2\left[\frac{1-\mu_{\tilde{x}_2}}{1-\mu_{\tilde{\alpha}_1}}\frac{\Delta\alpha_1}{\alpha_1}x_2 + \frac{1-\mu_{\tilde{x}_2}}{1-\mu_{\tilde{\alpha}_2}}\frac{\Delta\alpha_2}{\alpha_2}x_2\right] \tag{6.20}$$

Using the set in (6.11), the above expression becomes:

$$dV(x_1,x_2) = x_1\left[S_{\tilde{\alpha}_1}^{\tilde{x}_1}k_{11}x_1 + S_{\tilde{\alpha}_2}^{\tilde{x}_1}k_{22}x_1\right] + x_2\left[S_{\tilde{\alpha}_1}^{\tilde{x}_2}k_{11}x_2 + S_{\tilde{\alpha}_2}^{\tilde{x}_2}k_{22}x_2\right] \tag{6.21}$$

Letting $S_{\tilde{\alpha}1}^{e2} = 0 = S_{\tilde{\alpha}2}^{e1}$ and rearranging, (6.19) becomes:

$$dV(x_1,x_2) = [x_1\ x_2]\begin{bmatrix} S_{\tilde{\alpha}_1}^{x_1} & 0 \\ 0 & S_{\tilde{\alpha}_2}^{x_2} \end{bmatrix}\begin{bmatrix} k_{11} & 0 \\ 0 & k_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad or, \tag{6.22}$$

$$dV(x_1,x_2) = [x_1\ x_2]\begin{bmatrix} S_{\tilde{\alpha}_1}^{x_1}k_{11} & 0 \\ 0 & S_{\tilde{\alpha}_2}^{x_2}k_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \triangleq x^T P x \tag{6.23}$$

Then $dV(x_1, x_2) \leq 0 \Rightarrow$ P is negative definite ■

6.7 Generalization of the Robust Stability Result

In reality, a system may not be easily decoupled. Some interaction is bound to exist between certain modes of the system, especially at high frequencies. Even if this is not the case, sometimes certain modes of the system are just naturally coupled and that any attempt to decouple them is usually done only to simplify design or analysis. Such couplings may need to taken into account, depending on the particular system, its complexity and the required degree of operational precision. In the following, it is assumed that state interactions exist as a result of parameter variations. If indeed a given parameter, α_j, is considered to be a dominant characteristic of only the state x_j, then whenever α_j assumes a value other than its nominal value, one would expect, in general, a more than proportionate change in the $\dot{x}_j$ - dynamics than in the $\dot{x}_i$ - dynamics and vice versa, depending of course on how x_i and x_j are actually coupled. If this is not the case, then the interactions may not be assumed to be weak. The cross sensitivities must therefore be taken into account. These interactions should therefore be reflected in the P-matrix.

6.7.1 Stable Bounds of the Interactions

In the following, an expression for an approximate measure of the interaction will be derived in terms of the decoupled or state-characteristic unit perturbations k_{ii} and the sensitivities $S_{\tilde{\alpha}i}^{ei}$. Let us denote by ϵ_{ij} (or $S_{\tilde{\alpha}i}^{ej}$) the interaction that resulted because of the effect of varying α_i on x_j. In other words, it is the sensitivity function of e_j due to α_i. For n states, there will be $n^2 - n$ such interactions. This is reflected in the augmented matrix given below.

$$\begin{bmatrix} S_{\tilde{\alpha}_1}^{x_1} & 0 & \cdots & 0 \\ 0 & S_{\tilde{\alpha}_2}^{x_2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & S_{\tilde{\alpha}_n}^{x_n} \end{bmatrix} + \begin{bmatrix} 0 & \epsilon_{12} & \cdots & \epsilon_{1n} \\ \epsilon_{21} & \ddots & & \epsilon_{2n} \\ \vdots & & & \vdots \\ \epsilon_{n1} & \epsilon_{n2} & \cdots & 0 \end{bmatrix} = \begin{bmatrix} S_{\tilde{\alpha}_1}^{x_1} & \epsilon_{12} & \cdots & \epsilon_{1n} \\ \epsilon_{21} & S_{\tilde{\alpha}_2}^{x_2} & & \epsilon_{2n} \\ \vdots & & \ddots & \vdots \\ \epsilon_{n1} & \epsilon_{n2} & \cdots & S_{\tilde{\alpha}_n}^{x_n} \end{bmatrix} \tag{6.24}$$

The implication of this augmentation is that superposition can be used to determine the overall effect of the individual interactions. Specifically, $\epsilon_{i\,j}$ (due to α_j) can be determined by keeping $k_{j\,j} = \Delta\alpha_j / \alpha_j = 0$. In order to realize the full effect of the perturbations, the actual perturbations, $\Delta\alpha_i$ will be used instead of the per unit quantities, $k_{i\,i}$. Identical results are obtainable either way. The augmented matrix, $\tilde{P}$, in (6.22) becomes the total sensitivity matrix expressed as:

$$dV = [x_1 \cdots x_n] \begin{bmatrix} S_{\tilde{\alpha}_1}^{x_1} & \epsilon_{12} & \cdots & \epsilon_{1n} \\ \epsilon_{21} & S_{\tilde{\alpha}_2}^{x_2} & & \epsilon_{2n} \\ \vdots & & \ddots & \vdots \\ \epsilon_{n1} & \epsilon_{n2} & \cdots & S_{\tilde{\alpha}_n}^{x_n} \end{bmatrix} \begin{bmatrix} \Delta\alpha_1 & 0 & \cdots & 0 \\ 0 & \Delta\alpha_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & \Delta\alpha_n \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \tag{6.25}$$

$$\triangleq x^T \tilde{P} x, \quad where\ \tilde{P} = [p_{ij}]$$

On expanding, this becomes:

$$dV(x_1,\cdots,x_n) = \sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij}x_ix_j, \quad where:$$

$$p_{ij} = \begin{cases} S_{jj}\Delta\alpha_j, & i = j \\ \epsilon_{ij}\Delta\alpha_j, & i \neq j \end{cases} \tag{6.26}$$

For stability convergence in the presence of interactions between the states, $dV \leq 0$, gives: the set of inequalities below:

$$dV \leq 0 \Rightarrow x_j\Delta\alpha_j\epsilon_{ij}x_i + x_jS_{\tilde{\alpha}_j}^{x_j}\Delta\alpha_jx_j \leq 0, \ (with\ k_{ii}=0). \tag{6.27}$$

From these, the measures of the interactions are bounded as:

$$\epsilon_{ij} < -S_{\tilde{\alpha}_j}^{x_j}\frac{x_j}{x_i}, \quad \forall t. \tag{6.28}$$

The above set of inequalities will be revisited shortly. Meanwhile, let ϵ_{ij} be replaced by the respective infima of the right hand side of the inequalities as:

$$\epsilon_{ij}^{*} = \inf_{\alpha_j}\left[-S_{\tilde{\alpha}_j}^{x_j}\frac{x_j}{x_i}\right] \tag{6.29}$$

From section 6.6, the fuzzy sensitivity of e_j with respect to the parameter α_i may be expressed as:

$$S_{\tilde{\alpha}_i}^{x_j} \triangleq \frac{1-\mu_{e_j}}{1-\mu_{\tilde{\alpha}_i}} = \frac{1-\mu_{\tilde{\alpha}_j}}{1-\mu_{e_i}} S_{\tilde{\alpha}_j}^{x_j} S_{\tilde{\alpha}_i}^{x_i} \tag{6.30}$$

Using this and (6.29), the interactions are bounded as:

$$S_{\tilde{\alpha}_i}^{x_j} \leq \epsilon_{ij}^{*} \tag{6.31}$$

With these, the expression for *dV* from (6.25) becomes:

$$dV = [x_1 \cdots x_n] \begin{bmatrix} S_{\tilde{\alpha}_1}^{x_1} & \epsilon_{12}^{*} & \cdots & \epsilon_{1n}^{*} \\ \epsilon_{21}^{*} & S_{\tilde{\alpha}_2}^{x_2} & & \epsilon_{2n}^{*} \\ \vdots & & \ddots & \vdots \\ \epsilon_{nl}^{*} & \epsilon_{n2}^{*} & \cdots & S_{\tilde{\alpha}_n}^{x_n} \end{bmatrix} \begin{bmatrix} \Delta\alpha_1 & 0 & \cdots & 0 \\ 0 & \Delta\alpha_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & \Delta\alpha_n \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \tag{6.32}$$

$$\triangleq x^T \tilde{P} x, \quad where\ \tilde{P} = [p_{ij}]$$

The above performance function (6.30) needs to be minimized in order to obtain the 'best' values for $S_{\tilde{\alpha}i}^{xi}$ and $\Delta\alpha_i$ such that, in addition, $dV \leq 0$ for asymptotic stability convergence.

6.7.2 Robust Stability Bounds

The inequality bounds for the couplings have already been determined in (6.28) through (6.31). Thus the coupling terms are to be chosen such that:

$$\epsilon_{ij} \leq \epsilon_{ij}^{*}, \quad \forall t, \quad \forall\, i \neq j. \tag{6.33}$$

where the quantities in asterisks are the infima given in (6.29). An inherent restriction is that the sensitivities are not ideal thus making both interactions to be nonzero as desired. For a specified perturbation, $\Delta\alpha_i$, the corresponding interaction, ϵ_{ij} or ϵ_{ij}, can be determined upon knowing the profiles of the sensitivities. The infima of the interactions are given by:

$$\epsilon_{ij}^{*} \leq -S_{\bar{\alpha}_j}^{x_j}\big|_{\Delta\alpha_j} \, inf\left[\frac{x_j}{x_i}\right], \forall i,j,\ i \neq j,\ \forall t. \tag{6.34}$$

The bounds on the parameter perturbations are then obtained from the determinant conditions in (6.32). These bounds are not hard and fast even though they are written in precise inequalities. It may be difficult to satisfy them automatically, as the expression for dV is multi-parameter. Some heuristic procedure may have to be employed iteratively to resolve any conflicting requirements.

6.7.3 General Result for Robust Stabilization

A general version of the main result stated in Theorem 6.1 in section 6.6.1 now follows. Note that in general the $\tilde{P}$ is not symmetrical. Thus, in order to derive the general result, it needs to be transformed into a symmetric form. This is given below as:

$$dV(x_1,\cdots,x_n) = \sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij}x_i x_j, \quad where\ \tilde{P}|_{sym} = [p_{ij}]_{sym}\ with:$$

$$p_{ij} = \begin{cases} S_{jj}\Delta\alpha_j, & i = j \\ \dfrac{\epsilon_{ij}\Delta\alpha_j + \epsilon_{ji}\Delta\alpha_i}{2}, & i \neq j \end{cases} \tag{6.35}$$

Theorem 6.2:

The fuzzy logic controller will stabilize the whole of F in (6.4) without assumption 1 if the augmented matrix, $\tilde{P}|_{sym}$, in (6.31) is negative semi-definite, that is, if its leading principal minors are:

$$\Delta_1 = p_{11} = S^{x_1}_{\tilde{\alpha}_1}\ \Delta\alpha_1 \le 0, \quad \Delta_{12} = det\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \le 0,$$

$$\Delta_{123} = det\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \le 0, \ \ldots, \ \Delta_{12\ldots n} = \Delta(\tilde{P}_{sym}) \le 0 \tag{6.36}$$

6.7.4 Minimizing dV

Being the performance index, dV needs to be minimized. Now that $\tilde{P}$ is symmetric, a host of tools from matrix calculus can be applied. Suppose the errors are normalized as $\|x\|^2 = 1$. The minimization problem can be stated as $min\ x^T \tilde{P}\, x$ subject to $x^T x = 1$ or $(x^T x - 1) = 0$. This constrain can be adjoined to dV via a Lagrange multiplier, λ. The problem is then restated as:

$$\min(dV = x^T \tilde{P}_{sym} x) - \lambda(x^T x - 1). \tag{6.37}$$

This requires that the derivative with respect to each component x_i vanish. The result is a set of eigenvalue-eigenvector equations $Ax - \lambda x = 0$. If x satisfies this condition then

$$dV = x^T \tilde{P}_{sym} x) = \lambda x^T x = \lambda.\ (as\ x^T x = 1).$$
$$\therefore\ dV_{\min} = \lambda_{\min}.\ (and\ dV_{\max} = \lambda_{\max}). \tag{6.38}$$

Comment:

The conditions stated for robust stabilization by the FLC are all time-dependent and are therefore very strong; this is a consequence of the fuzzy sensitivities which are time-dependent. Thus, in general, dV will have to be monitored with respect to time, to observe its dynamic behavior. Also, from the matrix conditions, one obtains specific bounds in the form of inequalities on the desirable values of $\mu_{\tilde{x}}$ and $\mu_{\tilde{\alpha}}$ in their respective universes of discourse.

Even though one has been able to minimize the expression for dV, this does not tell us how to select the optimum sensitivities, perturbations and interactions. Without this minimization though, dV will be very conservative. So far, no bounds have been imposed on the sensitivities directly, other than the matix conditions. Ideally, it would be desired that each sensitivity be very small. In other words, it is undesirable that small parameter perturbations should lead to large trajectory errors. This then raises the question about the $\| \tilde{P}_{sym} \|$. Precisely, how large can the perturbations be before the FLC seizes to stabilize the system? Therefore, bounds on the sensitivity and the parameter variation needs to be derived in terms of inequalities. Before this is considered, some concepts pertaining to the ideal and worst case bounds on these quantities will be introduced, appropriately in fuzzy terms.

6.8 Fuzzy Extremes of Perturbations

The analysis being developed addresses the case of small parameter perturbation, $\Delta\alpha$, where

$$\Delta\alpha \in [-\Delta\alpha_m, \Delta\alpha_m], \quad \alpha_0 = \text{nominal}.$$

In terms of an arbitrary membership function shape, the fuzzy set for "Small" is illusttrated in Figure 6.4. Note that the origin in the universe of discourse corresponds to no parameter deviation, ie., $\alpha = \alpha_0$, the nominal value.

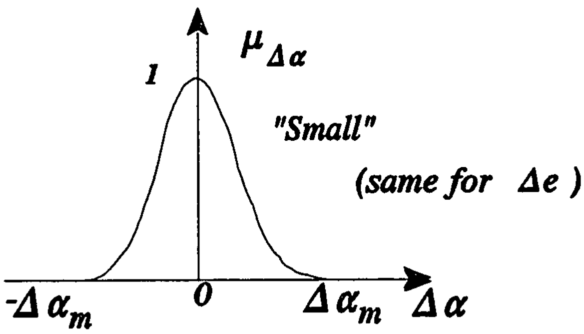


Figure 6.4 Fuzzy Set for Small Perturbations

Thus, any $|\Delta\alpha| > \Delta\alpha_m \neq$ "Small" and is not considered.

Definition 6.5: *Worst case Sensitivity* is such that a "small" parameter perturbation leads to the "largest" change in output. That is, if

$$(1 - \mu_{\Delta\alpha}) \to 0 \quad \Rightarrow \quad \mu_{\Delta\alpha} \to 1 \ni$$

$$(1 - \mu_{\Delta e}) \to 1, \text{ or } \quad \mu_{\Delta e} \to 0.$$

This is explained in the following illustrations (Figure 6.5).

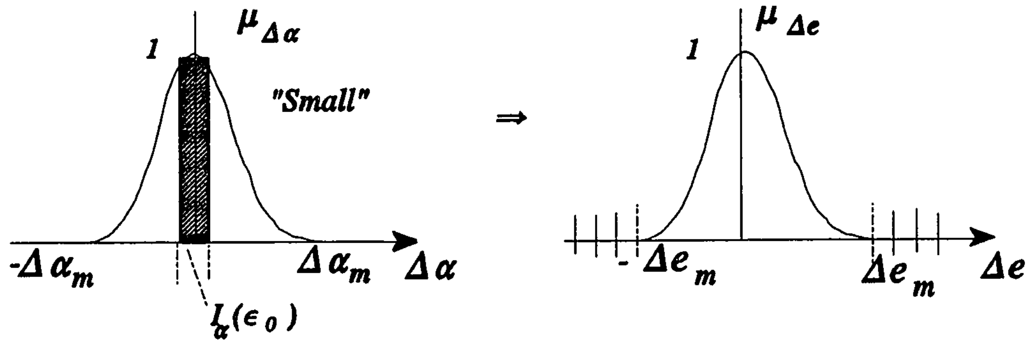


Figure 6.5 Worst Case Sensitivity

That is, If $\forall\Delta\alpha \in I_\alpha(\epsilon_0)$, $\mu_{\Delta\alpha} \rightarrow 1$ $\Delta e \in F(\Delta e) \setminus Supp(\mu_{\Delta e})$ such that

$1 - \mu_{\Delta\alpha} \rightarrow 0.$

- The result is therefore: $S_\alpha^e\, k_{ij} \rightarrow \infty$

Definition 6.7: *Ideal Sensitivity* is obtained when S_α^e is smallest, that is,

$$(1 - \mu_{\Delta e})/\ (1 - \mu_{\Delta\alpha}) \sim 0$$

In other words, the output level is unaffected by parameter perturbations. This situation is illustrated in terms of fuzzy quantities in figure 6.6.

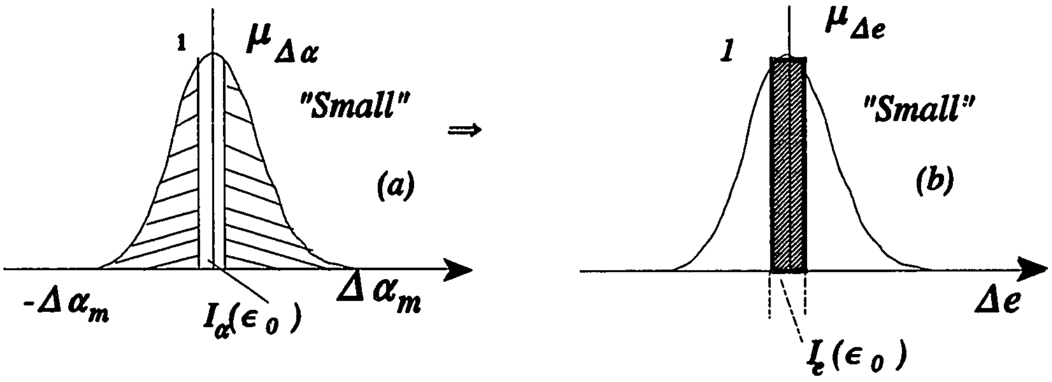


Figure 6.6 Ideal Case Sensitivity

The Condition for ideal sensitivity is such that if:

$\forall \Delta\alpha \in \text{Supp}\ (\mu_{\Delta\alpha}) \backslash I_\alpha(\epsilon_0) \Rightarrow \Delta e \in \text{a}\ I_e(\epsilon_0)$ on Δe, then

- The result is $S_\alpha^e\, k_{ij} \rightarrow 0$. (smallest or ideal case)

Key Robustness Questions:

- How large can k_{ij} be?
- How large can $S_{\alpha e}$ be?

- How large can $S_{\alpha e}\, k_{ij}$ be before the system is destabilized?

- ie., before $P = [\, S_{\alpha i}^{ej}\, k_{ij}]$ fails to be ≤ 0.

6.9 A Robustness Measure

The matrix conditions in (6.31) were obtained by applying Sylvester's rule for negative definiteness. The second condition is a determinant condition. However, it is known that the determinant of a matrix is not always a good indication of how near the matrix comes to being singular. The eigenvalues are hardly a better measure. A better measure of the near-singularity of a matrix is the set of singular values of the matrix. For the *real* matrix P, the singular values are given by:

$$\sigma_i = \left[\lambda_i\left(P^T P\right)\right]^{1/2} \tag{6.39}$$

These are nonnegative, real numbers ordered as: $\sigma_1 \leq \sigma_2 \ldots \leq \sigma_p$. The largest singular value is denoted by $\bar{\sigma}$ and the smallest, by $\underline{\sigma}$. The smallest singular value is a measure of singularity of a matrix. The smaller it is, the closer the matrix gets to being singular. A small $\underline{\sigma}$ is therefore an indication that the matrix is nearly singular. [85,86].

The Measure of Robustness

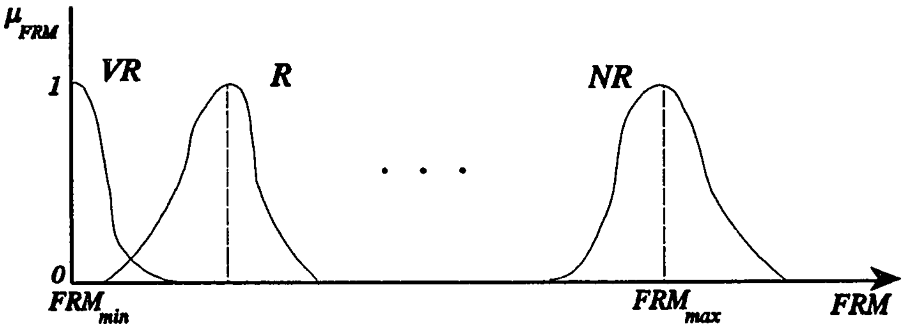
The supremum of the maximum singular value is gives a measure of the size of the augmented sensitivity matrix. This is given by:

$$\|\tilde{P}\|_\infty = \sup_t \bar{\sigma}(\tilde{P},t) \tag{6.40}$$

As a measure of robustness, the condition number of the matrix will be used when this is reasonably small, that is when the smallest singular value is different from zero. The condition number is defined as $\bar{\sigma} / \underline{\sigma}$, that is, the ratio of the largest to the smallest singular value. The larger this ratio is the faster the matrix approaches singularity. When an eigenvalue becomes very small, the condition number becomes excessively big and will not be used as the robustness measure, instead, the next smallest singular value will be used. For instance, in a two dimensional case, this is just the maximum singular value. Because these singular values are time varying, the robustness measure is obtained from above as the smallest of the ratio and is called the *Fuzzy Robustness Measure* (*FRM*) given by:

$$FRM = \frac{\sup_t \bar{\sigma}_i(\tilde{P},t)}{\inf_t \underline{\sigma}_i(\tilde{P},t)} = \frac{\| \tilde{P} \|_\infty}{\inf_t \underline{\sigma}_i(\tilde{P},t)} \qquad (6.41)$$

where $FRM \geq 1$. Consider the feasible universe of discourse for the fuzzy robustness measure to be $[FRM_{min}, FRM_{max}]$. The computed value of FRM in (6.39) can have membership in this interval which can be described linguistically in the fuzzy set in Figure 6.7.



VR = Very Robust, R = Robust, NR = Not Robust

Figure 6.7 Fuzzy Robustness Measure

As already mentioned, whenever $\underline{\sigma} \sim 0$, the *FRM* will be considered to be $\bar{\sigma}$. This will be required to be as small as possible for good robustness. The same linguistic description given in Figure 6.7 will still be used.

6.9.1 Comments

Various ways to derive inequality bounds on sensitivities and perturbations for robust stability were shown in this chapter. A lot of these do not directly impose any size restriction on the augmented sensitivity matrix. However, the use of singular values gives a measure of the size of this matrix. This, therefore, provides an additional measure that the designer may consider, in attempting to be as less conservative as possible, in the selection of the bounded design quantities. The *FRM* using (6.38) or (6.39) can also be used to get a "feel" for when the design is getting out of control, and therefore consider a possible redesign.

Redesign here refers to possibly doing with smaller perturbation in a particular parameter, or coping with a less than optimum sensitivity. Whatever the case, the designer should exercise engineering prudence in utilizing the hosts of inequality bounds in selecting the appropriate design parameters and the region of the universes of discourse of the relevant variables which provide a satisfactorily robust performance. The theory has been developed to include the case of both a coupled and a decoupled state interactions. By setting $\epsilon_{ij} = 0, \forall\, i \neq j$, the corresponding results under assumption 6.1 are obtained.

The matrix conditions are made possible because one has succeeded in putting dV in a quadratic form. This is only an unusual temporary comfort, so to say. In general, this may not be possible if the number of parameters is not the same as the number of states. The result is easily extended to include an external disturbance by considering this as one of the parameters. However, this decreases the possible number of parameters by one. If desired, the scalar expression for dV, instead of the matrix form, may still be used with a little loss of generality, and only some difficulty in achieving the explicit matrix conditions.

A Compromise between dV_{min} and FRM_{min}

When attempting to select the optimum parameters for robust performance, a compromise will have to be made between a high precision stability convergence to the set-point and what is merely an indication of a good performance behavior. Since dV is reqiured to be zero at the origin. Thus the closer to zero this is at steady state the better the convergence to the set-point. The minimum point of this does not necessarily correspond to the minimum value of the fuzzy robustness measure. The designer should therefore decide which of the options is more desirable for the situation.

The procedure for calculating the relevant fuzzy robustness quantities are outlined in Figure 6.8(a). Figure 6.8(b) explains the decision process for determining the linguistic robustness measures for the fuzzy control system.

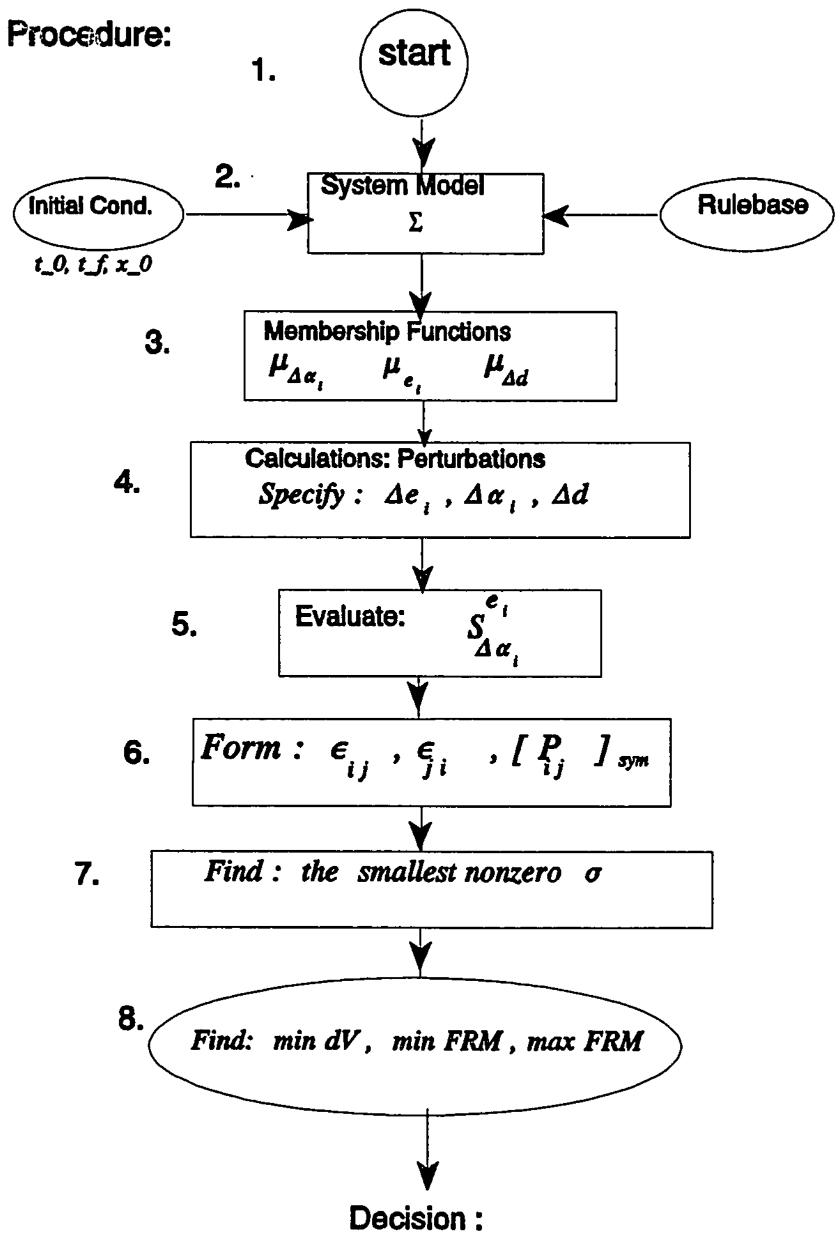


Figure 6.8(a) Procedure for Fuzzy Robustness Calculations

Decision :

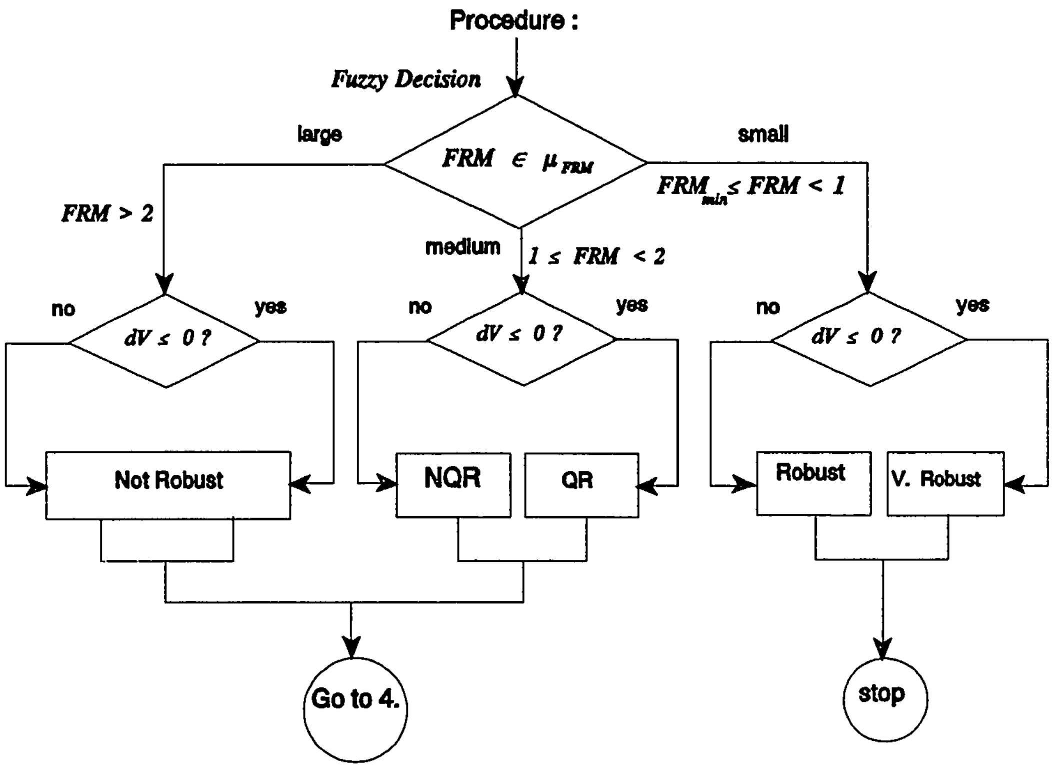


Figure 6.8(b) Decision Process for Determining FRM
(QR - Quite Robust, NQR = Not Quite Robust, V= Very)

6.10 Application Example

Consider the test-bed fuzzy idle speed controller for the two-state engine model in (5.10). The model is described by:

$$
\begin{aligned}
\dot{x}_1 &= k_p[a_0 x_2 + a_1 x_1 - a_2 x_1 x_2 - a_3 x_2 x_1^2 + g(x_1) a_4] + [k_p g(x_1) a_5] u_1 \\
&\quad + k_p[g(x_1) a_6 u_1^2] \triangleq F_1(x, u, \alpha_1, d) \\
\dot{x}_2 &= k_N[-a_7 + a_8 \frac{\dot{m}_{ao}}{x_2} + a_{12} x_2 - (a_{13} + a_{14}) x_2^2] + [a_{10} x_2 + a_{11}] k_N u_2 \\
&\quad + [-a_9 k_N] u_2^2 + a_{15} T_d \triangleq F_2(x, u, \alpha_2, d)
\end{aligned}
\tag{6.42}
$$

where $x = (x_1\ x_2)^T = (P\ N)^T$, $u = (u_1\ u_2)^T = (\delta\ \theta)^T$, $\alpha_1 = kp$, $\alpha_2 = kN$. The errors in P and N are given by $e_1 = x_1 - x_{1d}$ and $e_2 = x_2 - x_{2d}$; $d = T_d$ is a scalar disturbance. The general form of the linguistic rules for the controller in [61] was given as:

If (N is F_1) & (P is F_2) Then ($\Delta\delta$ is G_1) & (Δ is G_2).

where F_i and G_i are the consequent and premise linguistic terms respectively. For full detail refer to chapt. 3. and (61). In Chapter 3, the complete rulebase consisting of 56 rules was shown in Figure 3.6. It is instructive at this stage to show the same rulebase after clustering. This has reduced the number of rules to only 7 as shown in Figure 6.9. The result of this is a great reduction of computational burden in the analyses.

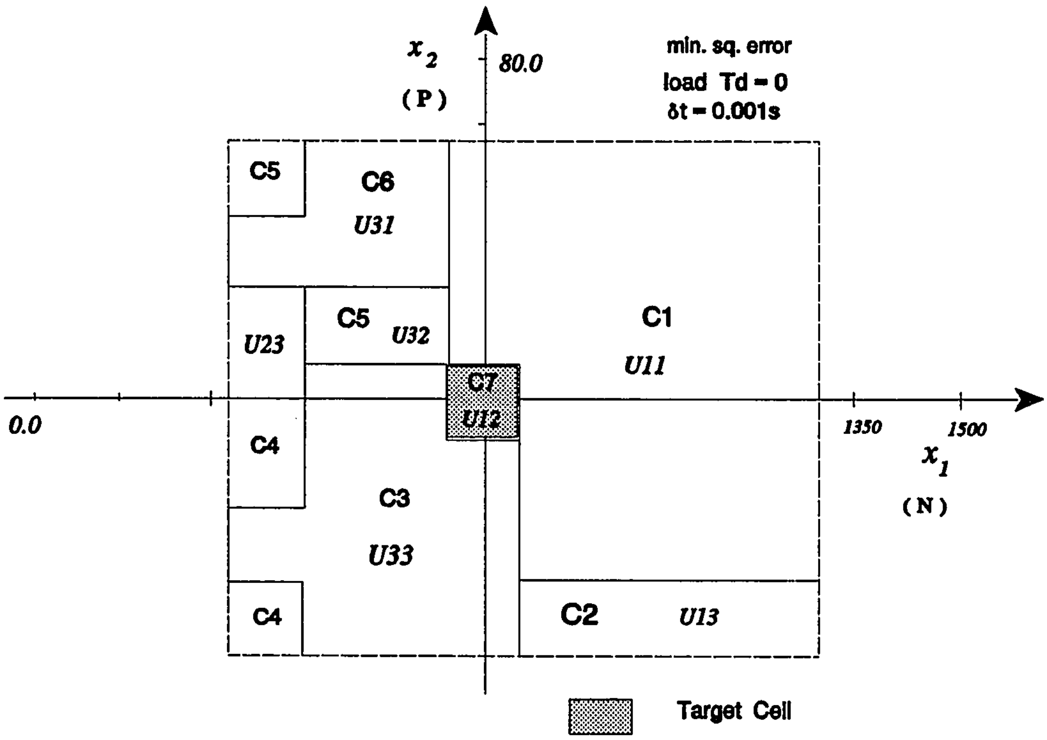


Figure 6.9 Clustered Rulebase for the Idle Speed Controller.

As a matter of interest, the above cell clustering is given below:

<u>Cell Clustering</u>:

C1: $814.151 \leq x_1 \leq 1500$ and $9.00 \leq x_2 \leq 80.0$
$689.226 \leq x_1 \leq 815.151$ and $38.995 \leq x_2 \leq 80.0$

C2: $814.5 < x_1 \leq 1500$ and $-0.996 < x_2 \leq 9.00$

C3: $439.377 < x_1 < 814.151$ and $-0.996 < x_2 \leq 28.998$
$439.377 < x_1 \leq 689.226$ and $28.998 < x_2 \leq 38.995$
$314.452 < x_1 \leq 439.377$ and $9.00 < x_2 \leq 18.990$

C4: $314.457 < x_1 \leq 439.377$ and $18.990 < x_2 \leq 48.994$
$314.458 < x_1 \leq 439.377$ and $-0.996 < x_2 \leq 9.00$

C5: $439.377 < x_1 \leq 689.226$ and $38.995 < x_2 \leq 48.995$
$314.458 < x_1 \leq 439.377$ and $58.992 < x_2 \leq 80.0$

C6: $439.377 < x_1 \leq 689.226$ and $48.994 < x_2 \leq 80.0$
$314.452 < x_1 \leq 439.377$ and $48.994 < x_1 \leq 58.992$

Target Cell, C7: $689.226 < x_1 < 814.151$ and $28.998 < x_2 < 38.995$

The fuzzy controls Umn have their original definitions as previously given in Chapter 3.

The application example will be illustrated for initial conditions in the regions defined by C1, C2 and C7, thus allowing allowing the firing of at least two fuzzy rules. For good robust performance under the parameter variations, the ouput deviation from the set-point should be in a small neighborhood of the center of cell C7. This acceptable neighborhood should be supplied by the designer. In this example, a speed deviation of less than 1% will be specified. This is not considerd too small at all. Even in the presence of a major disturbance such as the onset of an air conditioning unit (21 Nm), the speed varied only by about 3.5% of the set-point of 750 rpm (750 to 776 rpm), while a new pressure was maintained that was about 38% above the set-point of 34 kPa. However, to operate at this new higher set-point, an additional control authority was required. In the case of the parameter variations, it is required that all perturbations be such that an additional control activity will not be needed. In other words the fuzzy controller can hold its own up to a certain level of parameter variations, with all trajectories remaining in the specified small neighborhood of the set-point. This will be the general view-point taken in the theory developed, whether one is analyzing the robustness of an engine fuzzy controller or one for regulating the dynamic behavior of demand and supply about the equilibrium quantity and price, in a competitive market situation.

In the case of the test-bed fuzzy controller, $x \in \mathbf{R}^2$, $\alpha \in \mathbf{R}^2$, so $n = r = 2$. The parameters are multiplicative, each entering the system through a particular dynamics.

Nominal Values:

$\alpha_{10} = kp = 42.4$

$\alpha_{20} = kN = 54.26$

$x_{1d} = 34$, $x_{2d} = 750$, $d = 0$

"Small" Perturbations:

$\Delta\alpha_1$, $\Delta\alpha_2$ $\in [-11, 11]$

Set-point Deviation: $e_1 \in [-10, 10]$,(30%); $e_2 = [-12\ \ 12]$, (1.6%).

Problem statement:

To study the robustness properties of the fuzzy controller designed for the above nominal system with respect to parameter variations about their nominal values, and in a small neighborhood of the set-point.

The goal of the analysis is to be able to establish the best setting parameter for the most robust operation, ie, with the least set-point deviation. At this point also, the corresponding estimates of the measures of interactions and sensitivities are obtained. The analysis will proceed in the following manner.

1. Investigate assumption 6.1 in the chapter by varying α_1 with $\Delta\alpha_2 = 0$, and vice-versa, and observe the interactions.
2. Determine the best parameter variation for the most robust operations.
3. Determine the fuzzy robustness measure, by first determining FRM_{min} and FRM_{max}. For this, one needs only determine the FRM beyond which the set-point deviation is not acceptable, even though the system remains stable. At such a point, an additional control compensation may be needed as already discussed.
4. Select a set of values of the parameters, sensitivities and interactions during the "most robust" operation as found in 2.
5. Verify the system's operation by selecting parameters from the optimum set.

For $r = n = 2$, the indices i,j range from 1 to 2. The following are obtained:

$$dV = [x_1 \; x_2] \begin{bmatrix} S_{\tilde{\alpha}_1}^{x_1} & \epsilon_{12} \\ \epsilon_{21} & S_{\tilde{\alpha}_2}^{x_2} \end{bmatrix} \begin{bmatrix} \Delta\alpha_1 & 0 \\ 0 & \Delta\alpha_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{6.43}$$

$$\triangleq x^T \tilde{P} x, \quad where \; \tilde{P} = [p_{ij}]$$

The symmetrical $\tilde{P}$-matrix is immediately formed from above as:

$$\tilde{P}|_{sym} = \begin{bmatrix} S_{\tilde{\alpha}_1}^{x_1}\Delta\alpha_1 & \dfrac{\epsilon_{12}\Delta\alpha_2 + \epsilon_{21}\Delta\alpha_1}{2} \\ \dfrac{\epsilon_{12}\Delta\alpha_2 + \epsilon_{21}\Delta\alpha_1}{2} & S_{\tilde{\alpha}_2}^{x_2}\Delta\alpha_2 \end{bmatrix}, \quad with \; \sigma_i = \sqrt{\lambda_i(\tilde{P}^T\tilde{P})} \tag{6.44}$$

where the sensitivities are in given in the set below.

$$\left\{ S_{\tilde{\alpha}_1}^{e_1} = \frac{1-\mu_{e_1}}{1-\mu_{\Delta\alpha_1}}, \; S_{\tilde{\alpha}_2}^{e_2} = \frac{1-\mu_{e_2}}{1-\mu_{\Delta\alpha_2}}, t; \Delta\alpha_1, \Delta\alpha_2 \right\} \tag{6.45}$$

The interactions are given by:

$$\epsilon_{12} < -S_{\tilde{\alpha}_2}^{x_2} \frac{x_2}{x_1}, \; and \; \epsilon_{21} < -S_{\tilde{\alpha}_1}^{x_1} \frac{x_1}{x_2} \; \forall t. \tag{6.46}$$

$$note: \; e_i \triangleq x_i = x'_i - x_{id}$$

Membership Functions:

The intervals already specified for the perturbations will be considered as their universes of discourse. The linguistic labeling is done heuristically. A more systematic way to construct the membership functions for the errors is to use a probablistic means to convert the time-varying signals $e_i(t)$ to membership functions using the method of windowing. In this example the errors are assumed to be uniform in the "small" neighborhood of the set-point. This assumption may be valid if the set-point is the stable equilibrium for the system for as $t \to \infty$, the error transients become progressively of bounded and diminishing variations in this neighborhood. Thus, $\exists\, t_0$ such that corresponding to the interval $[\, t_0, t_\infty]$, is a sequence of positive, negative numbers, including the number zero, which converge uniformly to zero at the set-point. It is from this sequence that the membership functions for the "small" errors are derived. Among all possible shapes, the triangular membership functions are employed for ease of computation. These are shown in Figure 6.10.

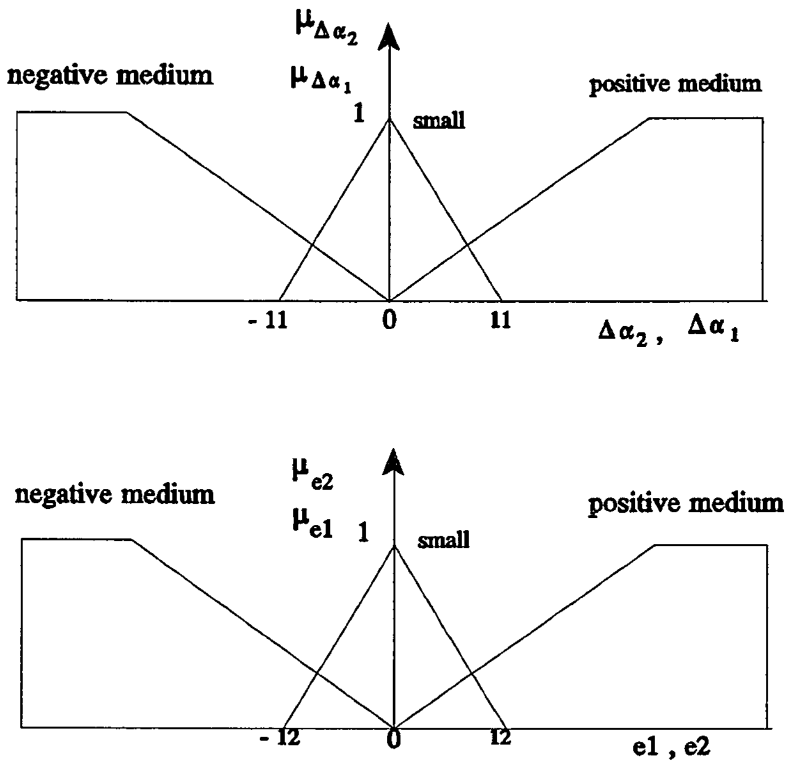


Figure 6.10 Membership functions for (a) $\Delta\alpha_i$ (b) e_i

6.10.1 Simulation Studies

Several simulations have been conducted with various combinations of the two parameters' variations. Variations that are both below and above the nominal values of these parameters have been considered. However, only a handful of representative cases are discussed here. The essence is to show particularly, that the relevant fuzzy quantities used in the formulation of the theory can be computed. In order to simplify the notation, the sensitivities S_{ii} are used in place of $S_{\ddot{\alpha}i}^{ei}$, for all $i.(i=1,2)$. In the first set of simulations (Figures 6.11-6.14), the parameters were varied such that α_1 was fixed at its nominal value, $\alpha_{10} = 42.40$, ie., $\Delta\alpha_1 = 0$, while α_2 was varied below its nominal value, $\alpha_{20} = 54.26$. The fuzzy controlled system was simulated for 2 seconds and the relevant quantities $S_{1\,1}(t)$, $S_{2\,2}(t)$, $\epsilon_{1\,2}(t)$, $\epsilon_{2\,1}(t)$, *dV/dt*, *FRM(t)* were computed and plotted as functions of time. The second set of simulations consisting of Figures 6.15-6.18 was essentially the same as the first set except that this time the parameter α_1 was varied by the same amount below its nominal value as in the first set while $\Delta\alpha_2 = 0$. The reason for this procedure was to observe if the interactions between the states might be neglected and the system considered decoupled under the effect of the parameter variations, which would allow a much simpler analysis. In the third set of simulations, Figures 6.19-6.23, an optimum fuzzy robustness set was sought which consisted of the individual sensitivities, the cross-sensitivities, *dV/dt* and *FRM* such that the set-point deviations are minimum. The last set of simulations (Figures 6.24 and 6.25) was conducted so as to address the issue of compromise between dV_{min} (or minimum steady state errors) and FRM_{min}. By further varying α_2 lower than in the third set, a slightly different robustness set was obtained. The *minimum* set of the optimum robust elements may described as:

$$\Re_\alpha = \{ S_{11}, S_{22}, \epsilon_{12}, \epsilon_{21}, \Delta\alpha_1, \Delta\alpha_2; dV, FRM\}$$

For the two cases (a) and (b) considered, the sets are:

$$\begin{aligned} \Re_{\alpha}:(a) &= \{0.10845, 1.1724E\text{-}2, 2.3883E\text{-}2, 5.3240E\text{-}2, \\ &\quad -5.10, -10.3888; -0.2452, 0.6751\} \\ \Re_{\alpha}:(b) &= \{0.10849, 1.1466E\text{-}2, 2.3356E\text{-}2, 5.3261E\text{-}2, \\ &\quad -5.10, -10.3889; -0.2434, 0.6726\} \end{aligned} \tag{6.47}$$

The justification as to whether to use parameters from the set (a) or (b) depends on the designer and the particular application. In particular, the question that needs to be answered is whether the slight authority in further varying α_2 from the value in (a) to the value in (b) is worth the better convergence of x_2 (Figures 6.21 and Figure 6.24) at the expense of a lesser robustness (higher *FRM*) that resulted. In both cases, the fuzzy control system is "Very Robust" (*VR*). Note that during all of these parameter variations, $\Delta\alpha_1 = -5.10$ was the 'best' found. Finally, the fuzzy controls for the regions tried in the simulations are shown in Figures 6.26 and 6.27. The results of the simulations are given next.

6.10.2 Simulation Results

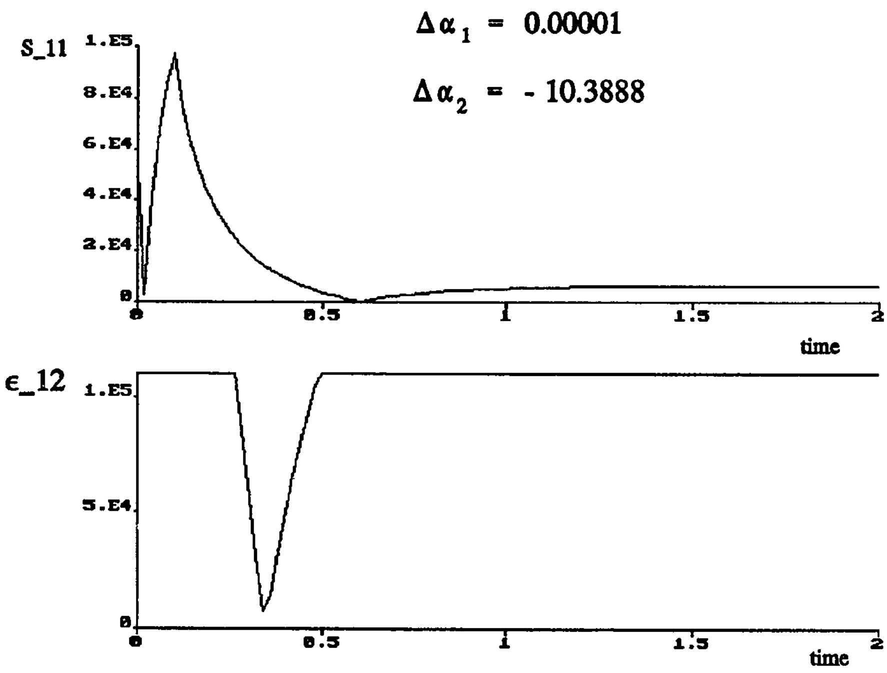


Figure 6.11 Sensitivity function (Top) and Cross Sensitivity (Bottom)

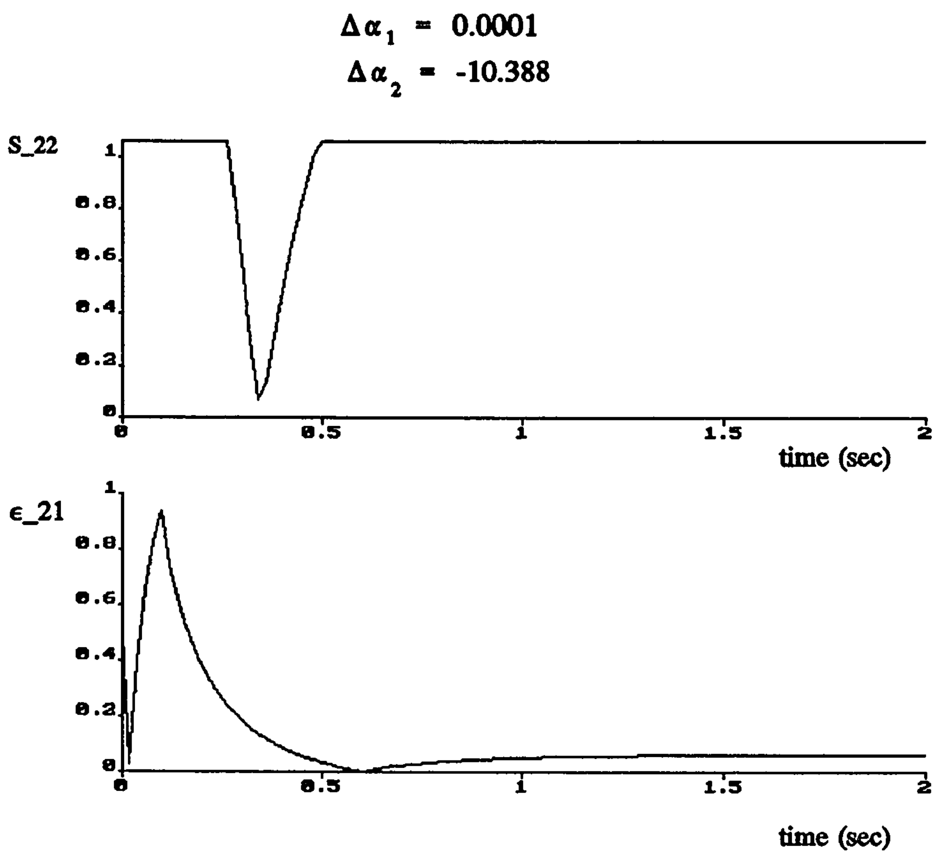


Figure 6.12 Sensitivity functions: (Top) S_22 (Bottom) ϵ_21

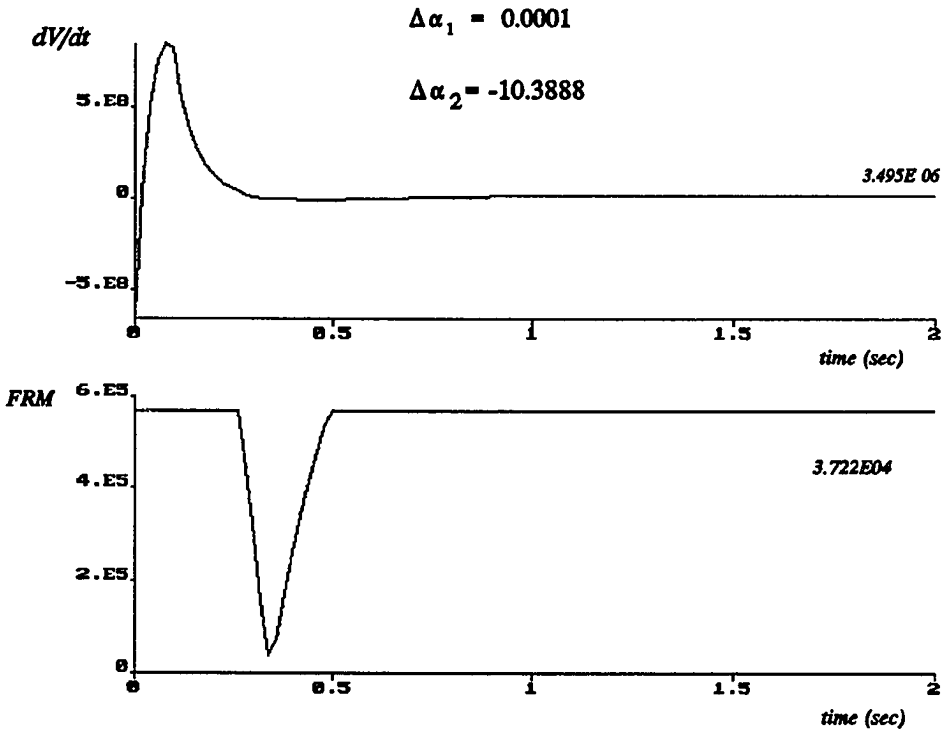


Figure 6.13 *dV* (Top) and Fuzzy Robustness Measure(*FRM*) (Bottom)

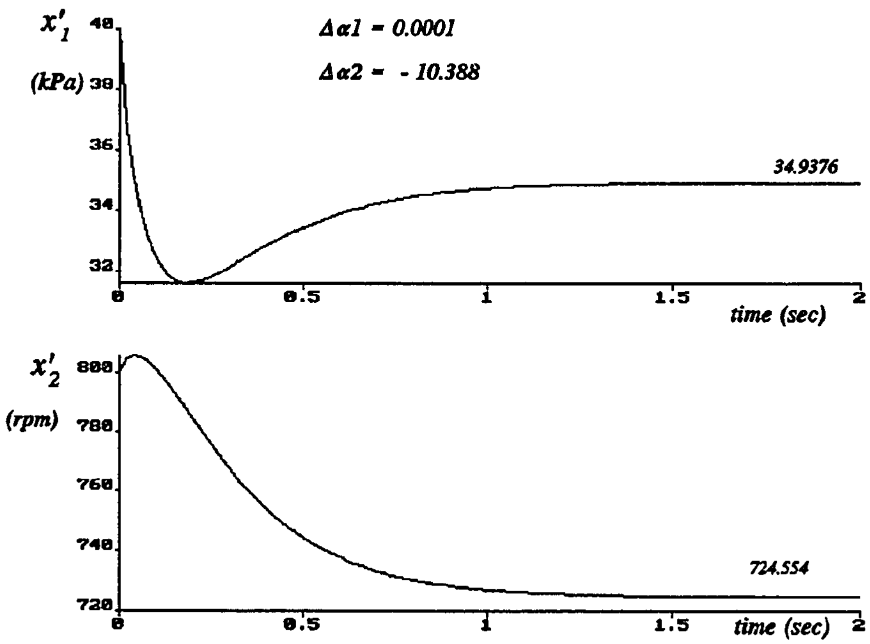


Figure 6.14 State Trajectories with only $\Delta\alpha_2$ varied

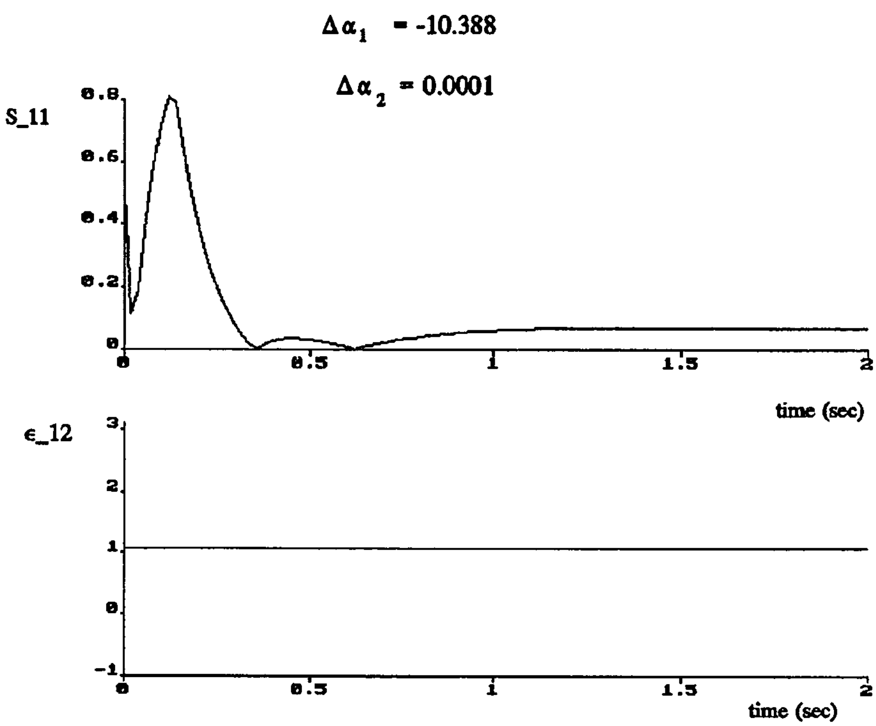


Figure 6.15 Fuzzy Sensitivity(Top) and Cross-sensitivity(Bottom) w.r.t. α_1

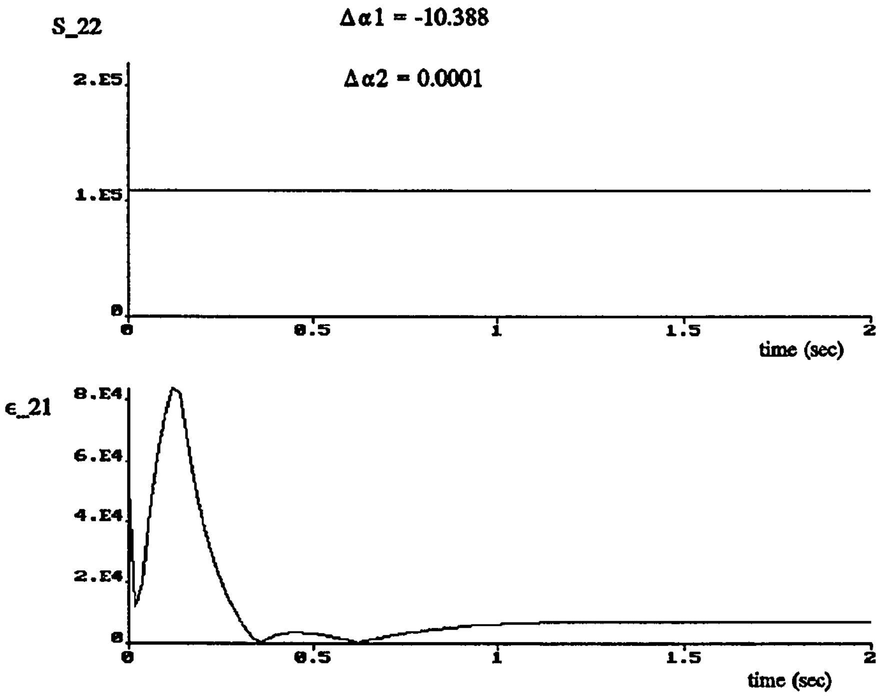


Figure 6.16 Fuzzy Sensitivity(Top) and Cross-sensitivity (bottom) w.r.t. $\Delta\alpha_1$

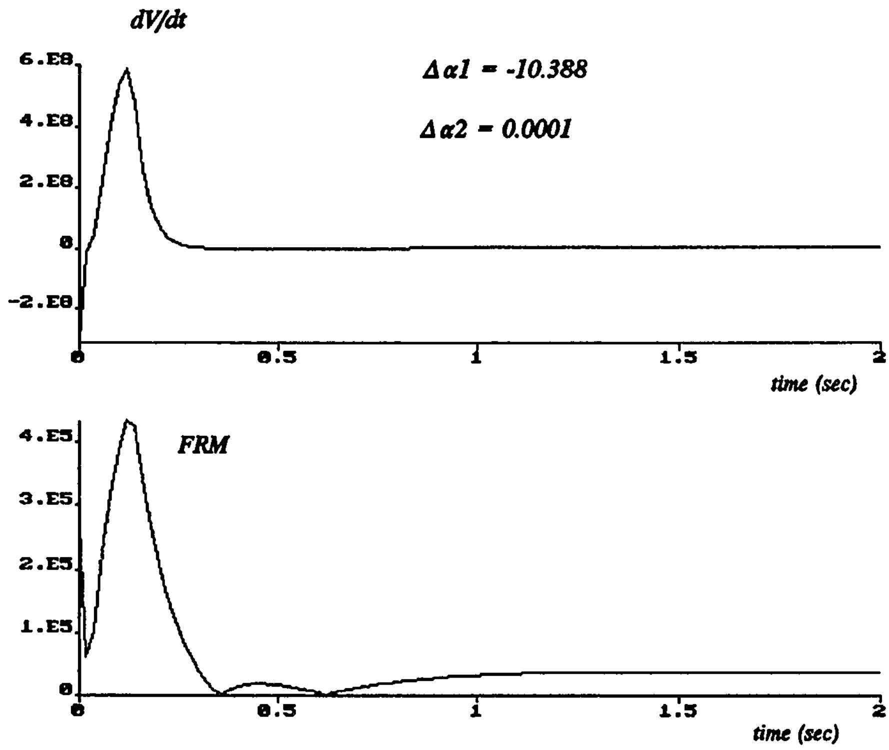


Figure 6.17 *dV/dt* and Fuzzy Robustness Measures w.r.t. $\Delta\alpha_1$

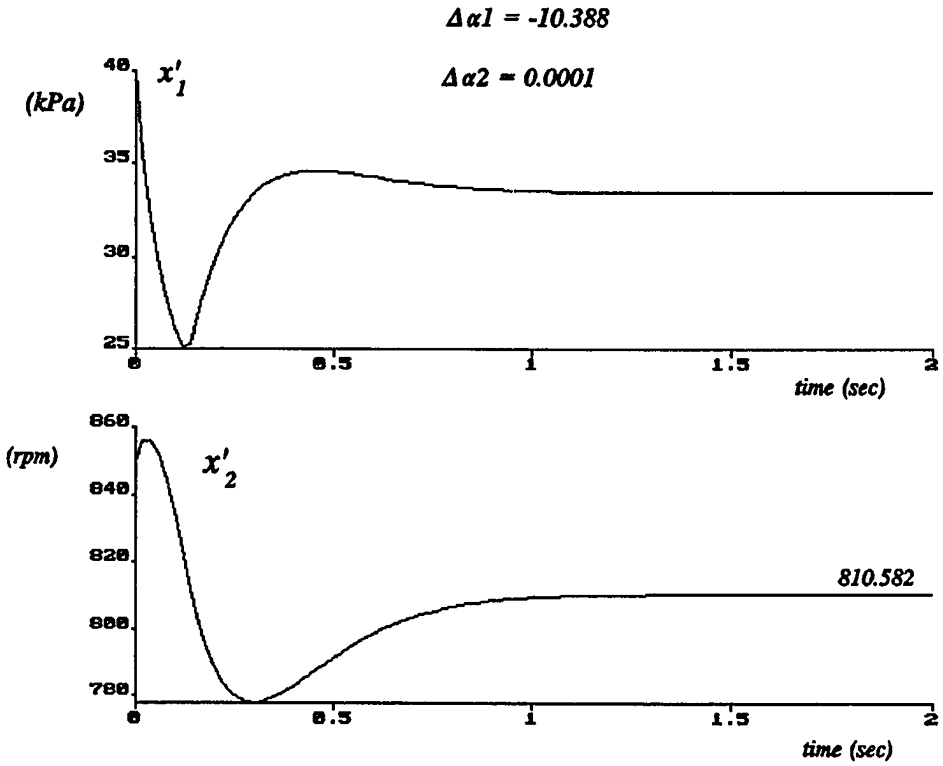


Figure 6.18 The actual State Trajectories: Pressure(top) and Speed(bottom)

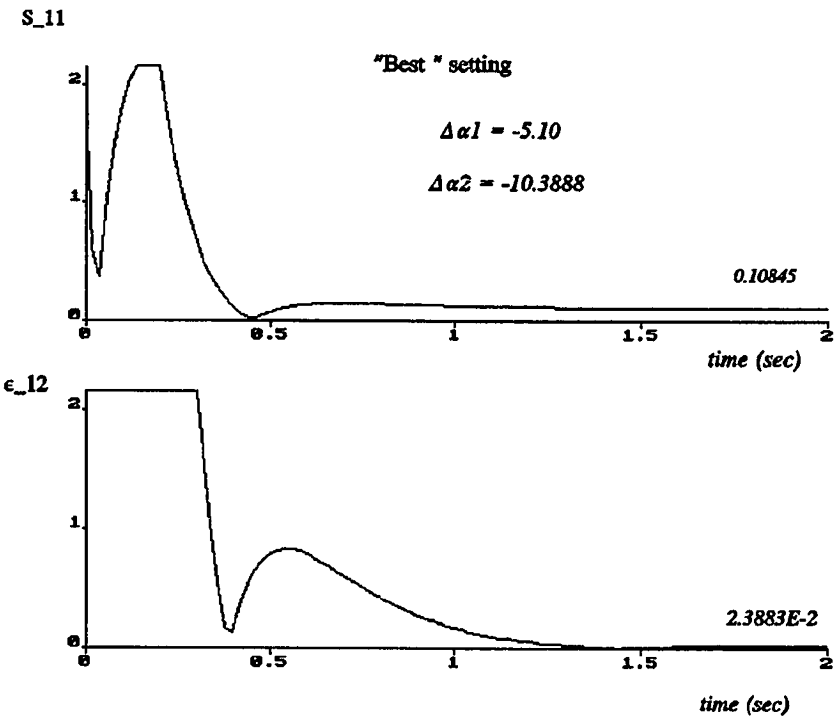


Figure 6.19 Sensitivities under 'Best' Setting (both parameters varied)

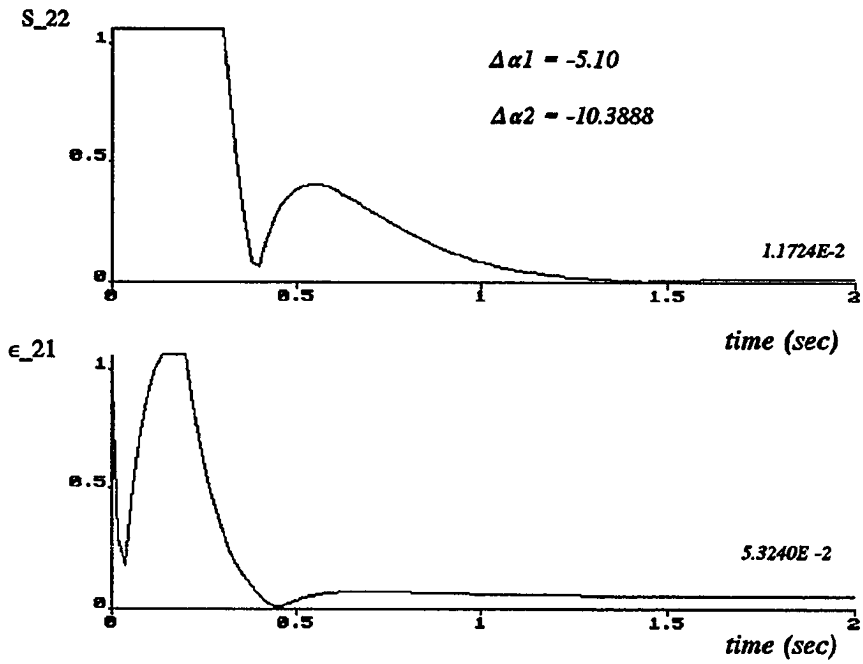


Figure 6.20 Sensitivities under "Best" setting (both parameters varied)

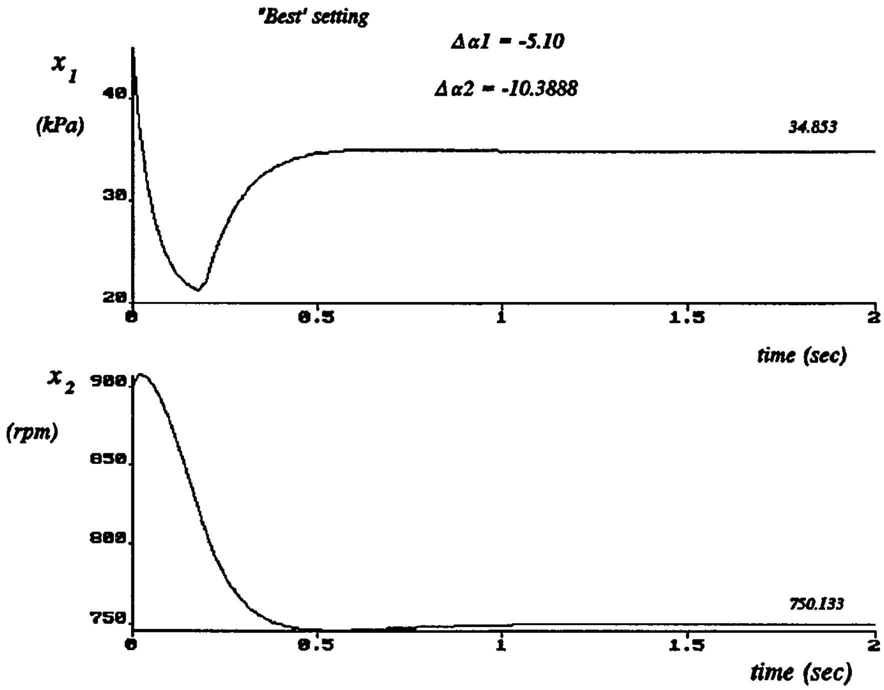


Figure 6.21 Stability convergence of State trajectories (both parameters varied)

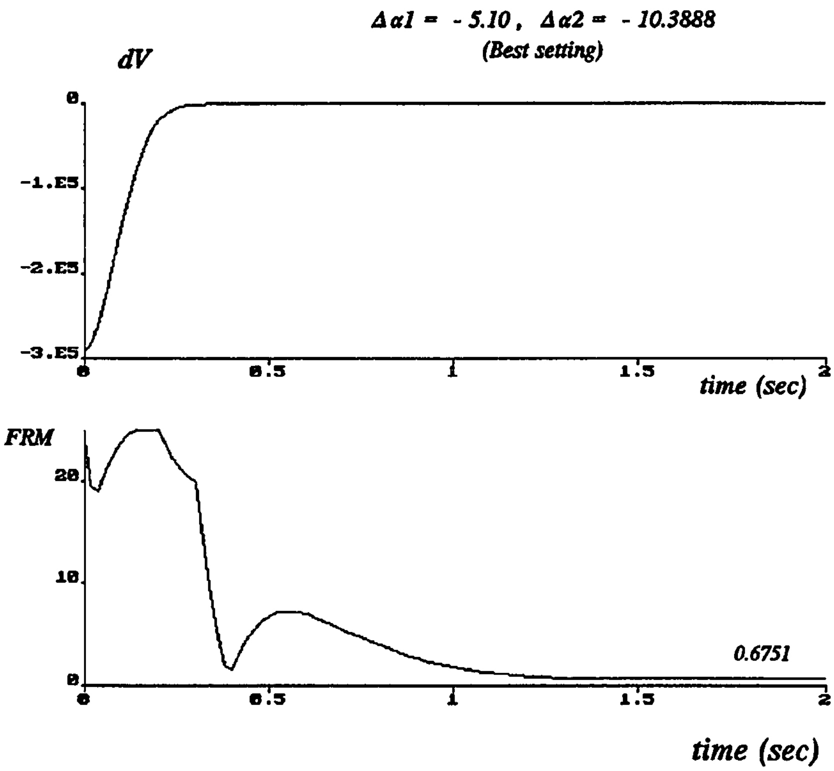


Figure 6.22 Plots *dV* and *FRM* (both parameters varied)

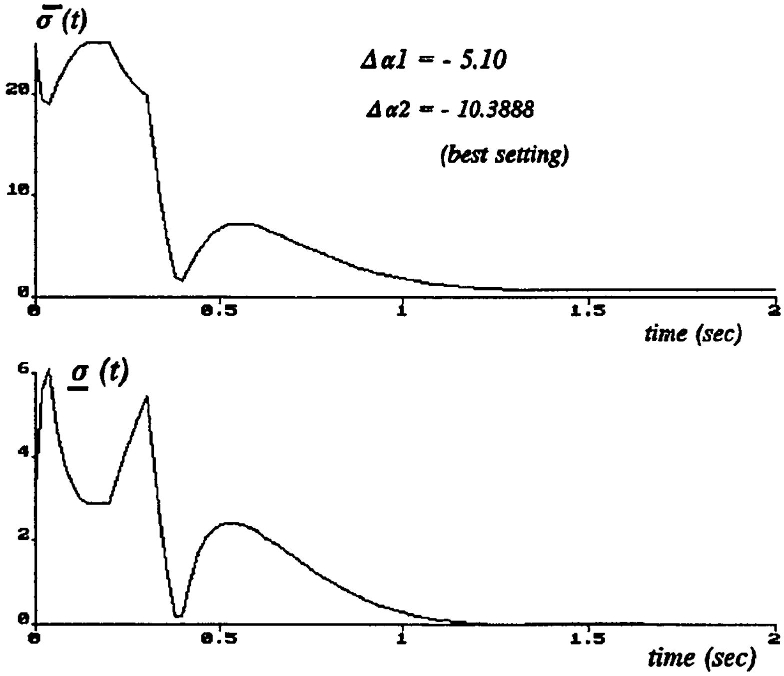


Figure 6.23 Singular Values "Best" setting (both parameters varied)

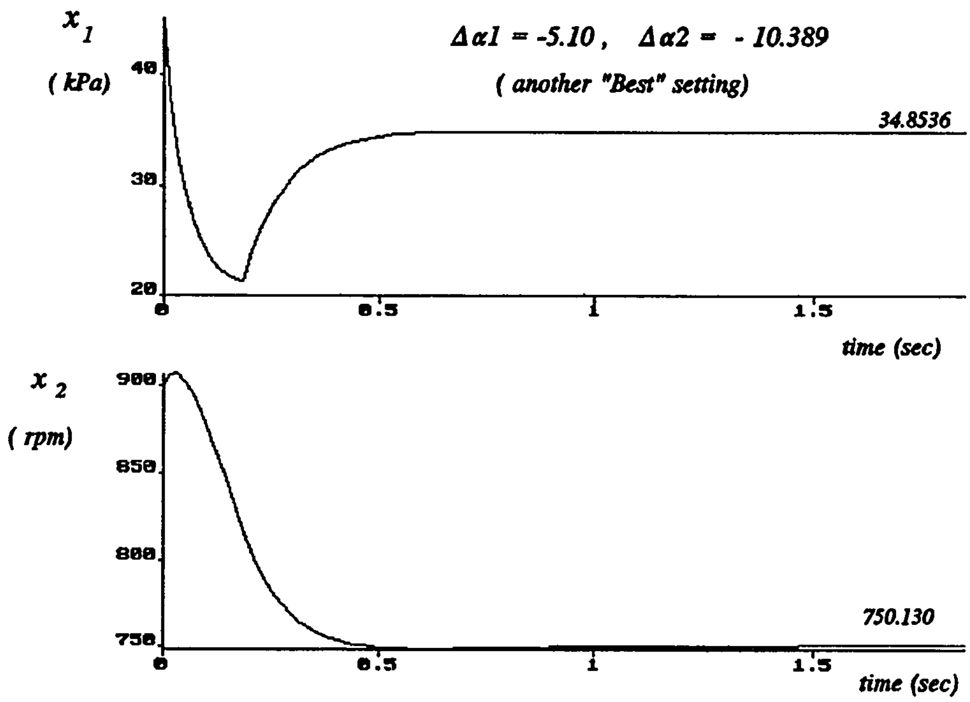


Figure 6.24 An Optimum Convergence of State Trajectories

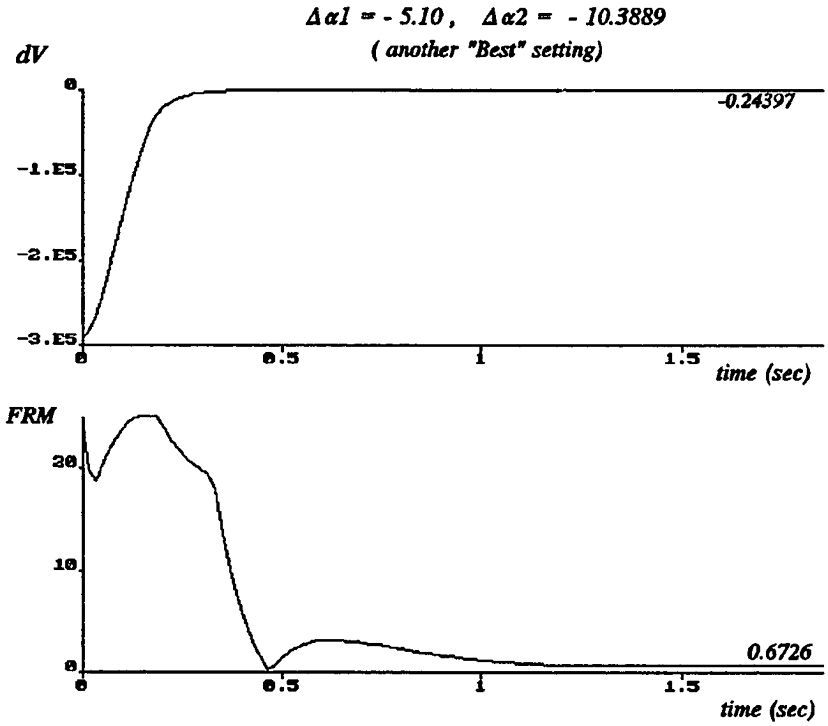
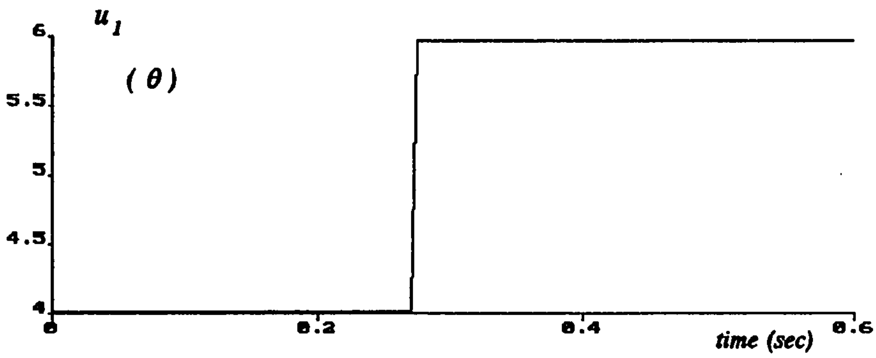


Figure 6.25 Plots *dV* and *FRM* for a different Optimum setting



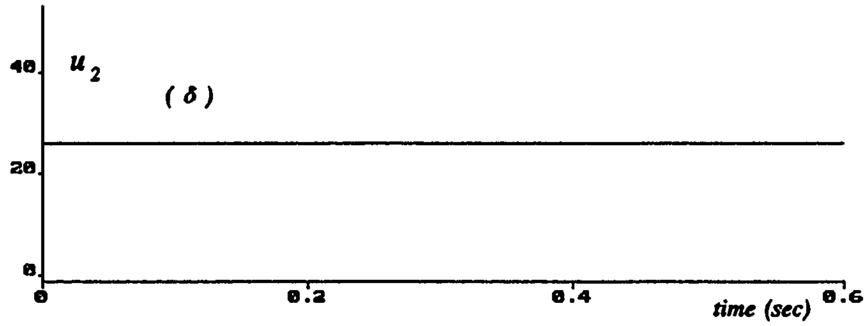


Figure 6.26 The Fuzzily Derived Controls in C1, C3, C7

6.10.3 Conclusions

The results of the simulations suggested that for this fuzzy control system, the parameters should be varied below their nominal values in order to attain a "very robust" performance. The variation of α_1 was in accordance with the stability requirement as stated in the theory, that $\Delta\alpha_1 < 0$. It was found that with α_2 fixed and α_1 varied, the effect on e_2 is $\epsilon_{12} = 1.0965$, was small but nonzero. Compared with the individual sensitivity $S_{11} = 6.9192 \times 10^{-2}$, $\epsilon_{12} / S_{11} = O(10^2)$ and not $\neq 0$. Thus ϵ_{12} may not be neglected. On the other hand, by keeping α_1 fixed, that is $\Delta\alpha_1 = 0$, and varying α_2, the effect on e_1 as measured by ϵ_{21} is 6.0677×10^{-2} while $S_{22} = 1.0589$. In this case ϵ_{21} / S_{22} is quite small. However, the decision on whether or not to neglect this time depends on the particular design requirement. Overall, the theory developed provides a systematic framework for analyzing the robustness of the fuzzy idle speed controller.

6.11 Summary

The theory introduced and developed in this chapter addresses the issue of robustness of an input-output map with respect to parameter perturbations, and external disturbances in general. The general notion of sensitivity and robustness were introduced followed by a statement of the robustness problem. The systematic formulation of the robustness measure developed here allows a class of both linear and nonlinear systems operating under fuzzy logic rules to be analyzed for robust performance. The result of the theorem gives the required inequalities on the sensitivities S_α^e and therefore on the membership function μ_e, and also on the allowable perturbations $\Delta\alpha$. Later on, an estimate of the coupling of parameters is first derived and a similar but more general result for robust performance is derived. Based on these, the sensitivity matrix, $\tilde{P}$, can be made as "tight" as possible using singular values such that the conclusion on dV continues to hold. A fuzzy control is considered the only stabilizing element in the closed-loop system. In the development, it is assumed that the system's states are available for measurement, and that the system's outputs are some of these states. In order to incorporate the prime design specification, the analysis has been developed in terms of the trajectory errors which needed to be minimum. The type of stability on question is therefore an asymptotic stability convergence of the bounded outputs, to the prescribed desired output, to within a small, finite, nonzero tolerance. This should hold for all bounded, continually fuzzily generated control inputs and a class of perturbations. Finally, the fuzzy robustness measure is formulated quantitatively, and is also expressible in linguistic terms such as robust, very robust, not robust, and so on. As an application example, the test-bed fuzzy idle speed controller for the automotive engine was used. The theory was seen to be a valuable tool for systematic robustness analysis for the class of fuzzy control systems considered. Though in the example robustness around the equilibrium point was analyzed, this could equally well be done in other regions of the state space where the system's robust operation is of prime concern. In such a case, membership functions are to be constructed for universes of discourse in the particular locality. Also, robustness to large parameter perturbations can be handled easily by heuristically determining what constitutes a "large" perturbation for the particular system. The final result in the analysis is the determination of the optimum robust parameter set or sets. The choice of the design elements from this set should therefore lead to a "very robust" design.

CHAPTER VII

SUMMARY AND CONCLUSIONS

Discussions and Summary

This dissertation addressed the issue of assessing the performance of fuzzy logic control systems. These are in the class of dynamical systems that are controlled via fuzzy logic and fuzzy sets. The concept of fuzzy sets was introduced by L.A. Zadeh in 1965, and about a decade later, it found applications in the design and control of systems. E.H. Mamdani and Assillian were the first to put this new concept to test on the control of a laboratory steam engine. Today, application domains have exploded enormously. The automatic train control, camera auto-focus, refrigerator control, electric ovens, washing machines, water treatment, waste management, aircraft, traffic control, are just a few of the applications of logic control. Recently, the auto industry in Japan has taken bold steps to incorporate fuzzy logic in a top of the line automobile's suspension and ride control and other automotive sub systems. Here, in the United States, some auto industries have already taken similar steps or are proposing to do so. There are reports of applications in climate control and anti-lock brake systems. The trends in fuzzy logic control seem upward in the years to come. Fuzzy chips, capable of processing thousands of fuzzy inferencing per second (FIPS) are already a common place, even though they are customized. This tends to open the door for a greater integration of fuzzy designs into the existing system architectures, either as supplements or standalones.

Basically, the reason why this huge embrace is taking place may be that, for systems whose control takes on a more or less humanistic form, it is considered more appropriate to describe the control process in an imprecise or fuzzy manner. On trying to be absolutely precise in the description of the design or the control process, one stumbles into the incompatibility principle that Zadeh alluded to. It was also found that such designs are, generally, fairly easily and much more quickly implemented. Another reason often associated with the embrace of the fuzzy control technology is that the controllers designed using a systematic methodology in particular, are, arguably, more flexible than their conventional counterparts. It is often argued that the gains of the fuzzy controllers are not fixed gains, but that they fall into a linguistic description of being "small", "very small", "negative large", "small near zero", "rarely large", etc., depending on the crisp values of the process variables. It is perhaps convincing to most that, if the system, by itself, can prudently place an incoming process data into the appropriate linguistic category or set, in a way that the system may humanistically be controlled, rather than handle it as being fixed, no matter what, then such a system should at least intuitively, possess a greater level of "robustness" when the incoming data is particularly imprecise. This imprecision in data is a norm rather than a rarity in the real world. Parameters drift due to aging, overuse, weather conditions, inappropriate parameter selections, and noise. It is therefore unwise to design the controller with fixed parameter and controller gains, just to return every so often to recalibrate these quantities, especially at some exorbitant consulting fee. Nevertheless, fuzzy logic control does not solve all of the control engineering problems. Contrary to the popular misconception, It is not supposed to do so. Indeed, fuzzy logic control is not without its misgivings and controversies. The misgivings arise for the fact that the original "tuning" of the controller could be painstaking. Essentially, this tuning process pertains to the judicious choice of membership functions, given only the design variables' universes of discourse. Tons of papers have been written on this subject. The controversies arose due to misconceptions about the structure of the fuzzy control paradigm. The earlier fuzzy control paradigms used essentially, and literally, the operator's manuals of the system to be controlled in order to derive the control rules, which are generally of the if-then type. It was then believed and for the most part asserted that a mathematical model of the process to be controlled was not needed in the control process. This immediately triggered a loud alarm, to say the least, as to the perceived voodooic nature of the new technique, in an era that was already blossoming with fantastic conventional and modern control techniques, all of which were model-based, and a lot of which had been proven, with an equally alarming success. The reported degree of apprehension and quite often, condemnation of the new technique and its "high priests" was particularly embarrassing, if not uninspiring. But then, the conventional control theorists and engineers were also in a quandary. The systems designed and controlled, without a mathematical model, using the operator manual-type if-then rules, performed just as well, and sometimes better than the systems that were designed using the prevailing control techniques. Obviously, there had to be a different plan of attack, for, on the ground of predicting the dynamic behavior of the controlled system, the two techniques ran neck-to-neck. In our research in fuzzy logic control, this new plan of attack was found to be both timely and appealing. It is essentially the following:

1) In the design of real engineered systems, the designer usually has an inkling of the knowledge of the process to be controlled. This knowledge could be in the form of a mathematical model of the process, no matter how crude or inaccurate. This was found to be essential for design, and more so for analysis purposes. This was the platform on which this thesis research was conducted.

2) So the operator manual-based if-then rules performed well in controlling the process, but how can their performance be assessed? In particular, how can the issue of

stability, robustness, controllability and like, be addressed and quantified?

These two plans seemed to turn the table on the early fuzzy controller designers. The work of Kizka on the energetistic stability of fuzzy systems was one of the early attempts to address the issue of stability; but even that work itself was non-model based. Indeed, recently, E.H. Mamdani apologetically recanted his every-where-quoted original claim that fuzzy logic control did not need a model od the process. Several model-based fuzzy systems design are being proposed in an effort to address the concern 2) above, in a manner that will appeal to both the main stream fuzzy aficionados and the modern control community. This is the attitude that was taken in the research efforts leading to this thesis. However, there is an eminent caution to be heed here. The fuzzy logic control is an entirely different way of designing a control system. The design involves heuristics, approximations, fuzzy logic and fuzzy sets, in addition, now, to the use of a mathematical model of the process, and certain control theoretic techniques as seen in the thesis. This notwithstanding, techniques in fuzzy logic control should not be equated, in an absolute sense, to similar techniques that are employed in a purely control theoretic setting. In the thesis, the performance measures considered of the closed-loop system are stability and robustness to parameter uncertainties. Because its intimate relationship with stability, controllability conditions for fuzzy systems was also developed. The developments of these performance measures has been more or less within the framework of certain known control theoretic concepts.

The major contribution of the thesis has been the robustness analysis of fuzzy control systems. The two stability theories developed are also significant contributions. The theory on the controllability of fuzzy systems is a contribution that deserves further development. In the stability analysis, a physical, engineered system was assumed. Because of this, the fact alluded to in the developments was that the energy of the stable system will gradually diminish as time progresses until a state of equilibrium is reached. Also, because rarely does a physical system in its useful regime, operate in "free" mode (zero inputs), the stability of the driven system was considered. A fuzzy controller in the closed-loop provided the inputs necessary to drive the process to its specified equilibrium state. Thus, not only should these control inputs be bounded, but also their applications should give rise to bounded process outputs. Thus, an input-output stability was considered. The first stability theory involves the formulation of an input-output mapping for the nonlinear or linear process. This mapping was then required to be made dissipative, strictly by the fuzzy logic controller. It was found out that there is no standard way of formulating the mapping for nonlinear systems. However, for a state-dependent system, this was formulated as an energy-like function of the states. A Stability condition was then imposed on the mapping for all points in the state space. If the controlled process is linear, an input-output mapping can be formed in a systematic way. In addition, if the process to be controlled is open loop stable, then the Kalman-Yakubovich (positive real) lemma could be applied to show that the system is dissipative under the action of the fuzzy control rules. The two cases were illustrated on two application examples: the test-bed fuzzy controller for the automotive engine and a missile autopilot. The second stability theory was formulated specifically for a class of nonlinear process set-point control systems. A heuristic argument was used to show that at least two dominating control actions could be identified in the control process. 1) A very coarse initial control activity when the process states are far off the set-point. This was associated with a relatively nonlinear behavior at the far-of states. As the system approaches, if it does, the vicinity of the set-point, the control activity becomes finer and finer and may be considered practically uniform, until the system converges to the desired set-point. With these settings, the dynamics of the system were decomposed into a linear part and a nonlinear residual. The fuzzy controller is to make this nonlinear residual be of order zero for all control and all initial conditions. If this happens, the heuristic argument holds, and asymptotic stability is guaranteed if the FLC also leads to a dissipative mapping of the linear part around the set-point. This is a new way of analyzing the stability of a process set-point control systems. The theory was demonstrated on the decomposed dynamics of the nonlinear automotive engine model under the test-bed fuzzy controller. In all cases, the results showed that the fuzzy control systems were stable. In particular, the stability tools developed can be used to effectively prove the stability claim of a fuzzy rulebase.

The robustness analysis is also new; it addressed the issue of robustness via the pertinent fuzzy quantities namely, membership functions and fuzzy rules. The theory is applicable to linear and nonlinear systems. The interesting thing is that even for nonlinear systems, a matrix expression is formulated on which to study the robustness properties. A performance index which was to be minimized, was specified as an energy-like function of the system's states. The results of this minimization were the relevant inequality bounds on the perturbation sizes, the profile of interaction strengths between states, and bounds on the various sensitivities. The theory proved very valuable in determining for instance, if a certain state-interaction or coupling may be neglected. This would render the analysis much simpler than it would otherwise be. At the end of the analysis, a fuzzy robustness measure was formulated using singular values. It was found that the most robust performances were obtained when the fuzzy robustness measure (FRM) was less than unity. However, as is usual with engineering practice, a compromise of some sort needs to be made when trying to select optimum design parameters. In the example considered, it was found that the smallest FRM did not necessarily correspond a perfect convergence to the desired set-point, or zero performance index. It turned out that a slightly higher FRM, which was nonetheless less than one, led to a better convergence to the set-point. This compromising situation should be reconciled with by the designer, all other things considered. The robustness analysis would be most suited in analyzing the robust behavior of model-based fuzzy control systems.

In the research here at Georgia Tech, a systematic fuzzy logic design methodology has been developed, and this tool should prove useful for analyzing fuzzy control systems designed using this or similar design methodologies. The results from the application example considered looked promising.

Future Extensions

The stability and robustness tools developed in the thesis have been used strictly for the purpose of analysis of fuzzy logic control systems. It would be very helpful if they can also be extended to be used for redesign of fuzzy rules, or the modification of membership functions. For example, where a system has shown an instability tendency, it would be very helpful to be able to do some "reverse" engineering so that the stability in the particular region alone might be improved. The starting point could be the value of dV/dt in the region of concern. This is particularly practicable in the case of the developed robustness analysis tool which provides bounds on the relevant design quantities. Finally, the controllability analysis developed in conjunction with the stability analysis could be further developed. For instance, there seems to be a natural dependence of the controllability conditions on the cardinality of the of the fuzzy sets of the system's variable. This dependency should be investigated and formalized. Also, the fuzzy companion form introduced in developing the controllability conditions, though not unique, could be employed in system identification of, particularly, a system described only by input-output data.

APPENDIX I

FUNDAMENTALS OF FUZZY LOGIC

I.1 Fuzzy Sets

Let U be a collection of objects denoted by {u}.

U $\triangleq$ Universe of discourse, which could be discrete or continuous.

u $\in$ U is a generic element.

Definition I.1: A fuzzy set F, in a universe of discourse U, is characterized by a membership function μ_F which takes values in the interval [0,1] namely, μ_F: U $\rightarrow$ [0,1]. This is a sharp contrast from traditional set theory where, given a set A and an element x, then either $x \in A$ or $x \notin A$. Membership here is binary, {0,1}. A fuzzy set may therefore be viewed as a generalization of the binary set. We represent a fuzzy set F in U as a set of ordered pairs of a generic element u and its grade of membership function,

$$F = \{(u, \mu_F(u)) | u \in U\}$$

When the universe of discourse U is continuous this is written as:

$$F = \int_U \mu_F(u)/U.$$

For a discrete universe of discourse, this is written as:

$$F = \sum_{i=0}^{n} \mu_F(u_i)/u_i$$

Example I.1:

Let the fuzzy set $F \triangleq \{$ *Integers close to 10* $\}$. If we restrict the elements of this set to:

$$\{ 7,8,9,10,11,12,13 \},$$

and heuristically let the membership set of these elements to be:

$$\{.1,.5,.8,1.0,.8,.5,.1\},$$

then using definition 1, we express the fuzzy set as:

$$F = \{(7,0.1),(8,0.5),(9,0.8),(10,1.0),(11,0.8),(12,0.5),(13,0.1)\}$$

or $$F = .1/7 + .5/8 + .8/9 + 1/10 + .8/11 + .5/12 + .1/13,$$

since the universe of discourse is discrete.

Definition I.2: *Support*: The support of a fuzzy set F, S(F) is the crisp set of all points $u \in U$ such that $\mu_F(u) > 0$. That is:

$$S(F) = \{ u \in U \mid \mu_F(u) > 0 \}.$$

Definition I.3: *Crossover Point*: The element $u \in U$ at which $\mu_F = 0.5$ is called the crossover point.

Definition I.4: *Singleton*: A fuzzy set whose support is a single point in U with $\mu_F = 1.0$ is called a fuzzy singleton.

Example I.2:

Let $U = \{1,2,3,4,5,6\}$. The membership function of some fuzzy set F is shown in Figure I.1

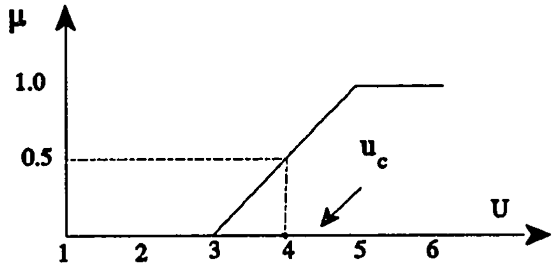


Figure I.1 Support and Crossover point.

Here the support is *{4,5,6}* and the crossover point is u_c. Note that even though the universe of discourse has six elements, the support has only three.

Definition I.5: *α-cut, strong α-cut*: the crisp set of elements that belong to the fuzzy set F at least to the degree α is called the *α-level set* or *α-cut*. This expressed as:

$$F_\alpha = \{ u \in U \mid \mu_F(u) \geq \alpha \}.$$

Stong α-cut, F_α' results when $\mu_F(u) > \alpha$ holds.

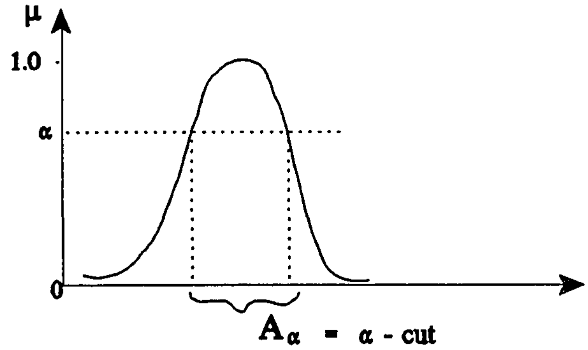


Figure I.2 α - Cut

Example I.3:

In example I.1 we define the fuzzy set F $\triangleq$ { Integers *close* to 10 }. The following α-cuts are obtained.

$$F_{0.3} = \{8,9,10,11,12\}$$

$$F_{0.7} = \{9,10,11\}$$

$$F_{1.0} = \{10\}$$

I.1.1 Basic Set Operations for Fuzzy Sets

As already explained, a fuzzy set is denoted in terms of its membership function. Set operations of fuzzy sets are therefore defined in terms of membership functions. The definitions employed here are those adopted by Zadeh in [1].

Consider the fuzzy sets A,B in some universe of discourse U.

Definition I.6: *Intersection*: The intersection (*and*) of A and B denoted by $A \cap B$ is pointwise defined by:

$$A \cap B \triangleq \int_U (\mu_A(y) \wedge \mu_B(y))/y, \text{ where } \wedge = \text{min operator}, y \in U.$$

Definition I.7: *Union*: The union (*or*) of A and B denoted by $A \cup B$ is pointwise defined by

$$A + B \triangleq \int_U (\mu_A(y) \vee \mu_B(y))/y, \text{ where } \vee = \text{max operator}, y \in U.$$

Example I.4:

If $U = 1 + 2 + \ldots + 10$

$A = .8/3 + 1/5 + .6/6$

$B = .7/3 + 1/4 + .5/6$

then $A \cap B = .7/3 + .5/6.$

and $A + B = .8/3 + 1/4 + 1/5 + .6/6.$

Definition I.8: *Complement*: The complement of A (*not A*) is denoted by $\neg A$ and is defined by

$$\neg A \triangleq \int_U (1 - \mu_A(y))/y.$$

Definition I.9: *Product*: The product of A and B is denoted AB and is defined by

$$AB \triangleq \int_U \mu_A(y)\, \mu_B(y)/y.$$

Example I.5:

If $A = .8/2 + .9/5$ and $B = .6/2 + .8/3 + .6/5$ then

$AB = .48/2 + .54/5.$

Definition I.10: Based on definition 9, given a positive number α and a fuzzy set A, we define A^α by

$$A^\alpha \triangleq \int_U (\mu_A(y))^\alpha/y.$$

Definition I.11: For any nonnegative real number α and a fuzzy set A, we define αA by

$$\alpha A \triangleq \int_U \alpha\, \mu_A(y)/y.$$

Definition I.12: *Concentration*: The operation of concentration is defined by

$$CON(A) \triangleq A^2$$

Definition I.13: *Dilation*: the operation of dilation is defined by

$$DIL(A) \triangleq A^{0.5}.$$

The effect of this operation is the opposite of that of concentration.

Definition I.14: *Cartesian Product*: If $A_1,\ldots,A_n$ are fuzzy sets in $U_1,\ldots,U_n$, respectively, the cartesian product of $A_1,\ldots,A_n$ is a fuzzy set in the product space U_1 x ... xU_n with membership function

$$\mu_{A1x\ldots xAn}(u_1,u_2,\ldots,u_n) = \min\{\mu_{A1}(u_1),\ldots,\mu_{An}(u_n)\}.$$

Definition I.15: *Fuzzy Relation*: An n-ary fuzzy relation is a fuzzy set in U_1 x ... xU_n and is expressed as $R_{U1x\ldots xUn} = \{((u_1,\ldots,u_n), \mu_R(u_1,\ldots,u_n)) \mid (u_1,\ldots,u_n) \in U_1 \text{ x} \ldots \text{ x } U_n\}$.

Definition I.16: *Sup-Star Composition*: If R and S are fuzzy relations in U x V and V x W, respectively, the composition of R and S is a fuzzy relation denoted by R o S and defined by:

$$R \circ S = \{[(u,w), \sup_v(\mu_R(u,v) * (\mu_S(v,w))], u \in U, v \in V, w \in W\}$$

where * could be any operator in the class of triangular norms, namely, minimum, algebraic product, bounded product, or drastic product [3]. In this thesis, the sup-star composition 'o' is only a max-min operation because we will be dealing with finite sets.

Example I.6:

If U_1 = {TOM,DICK} and U_2 = {HARRY,JIM}, the cartesian product is given by:

$U_1 \times U_2$ = { (TOM,HARRY),(TOM,JIM),(DICK,HARRY),(DICK,JIM)}.

Consider the binary fuzzy relation of *resemblance* between members of U_1 and U_2. Let this be expressed as

resemblance(R) =

0.8/(TOM,HARRY)+0.6/(TOM,JIM)+0.2/(DICK,HARRY)+0.9/(DICK,JIM).

We can also represent this as a relation matrix, R where

$$R \triangleq \begin{array}{c} \\ TOM \\ DICK \end{array} \begin{array}{c} \begin{array}{cc} HARRY & JIM \end{array} \\ \begin{bmatrix} 0.8 & 0.6 \\ 0.2 & 0.9 \end{bmatrix} \end{array}$$

Suppose we have another resemblance relation, S from U_2 to U_3 = {KRYS,SUE} expressed as:

resemblance(S) =

0.1/(HARRY,KRYS)+0.9/(HARRY,SUE)+0.1/(JIM,KRYS)+0.1/(JIM,SUE).

The relation matrix S for this is:

$$S \triangleq \begin{array}{c} \\ HARRY \\ JIM \end{array} \begin{array}{c} \begin{array}{cc} KRYS & SUE \end{array} \\ \begin{bmatrix} 0.1 & 0.9 \\ 0.1 & 0.1 \end{bmatrix} \end{array}$$

We can form the composition R o S as:

$$R \circ S = \begin{bmatrix} 0.8 & 0.6 \\ 0.2 & 0.9 \end{bmatrix} \circ \begin{bmatrix} 0.1 & 0.9 \\ 0.1 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.8 \\ 0.1 & 0.2 \end{bmatrix}$$

Definition I.17: *Fuzzy Inclusion* of two fuzzy subsets A and B of U is stated as

$$A \subseteq B \text{ if and only if } \int_U \mu_A(x)/x \leq \int_U \mu_B(x)/x.$$

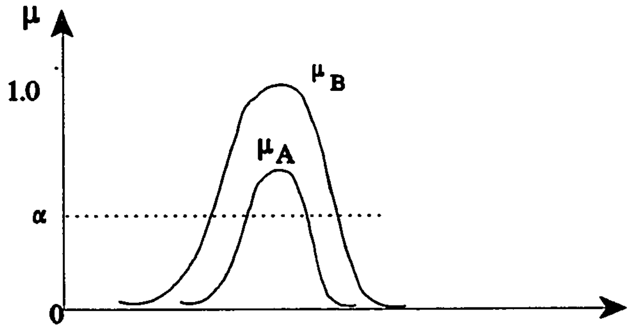


Figure I.3 Fuzzy Inclusion

Definition I.18: *Fuzzy equality* of two fuzzy subsets A and B of U is stated as

$$A = B \text{ if and only iff } \mu_A(x) = \mu_B(x), \ \forall x \in U.$$

I.2 Linguistics

Because of the novel nature of fuzzy logic as applied to control systems, it is felt only a matter of conscience to give an elaborate treatment of the fundamentals via definitions and examples of essential concepts and terminologies. This will enable a facile digestion of the various subject matter in this thesis in particular, and in the literature in general.

Definition I.19: *Fuzzy number*: A fuzzy number F in a continuous universe U, for example, the real line, is a fuzzy set F in U which is normal and convex. That is:

$$\max \mu_F(u) = 1, \ u \in U. \ \text{(normality)}$$

$$\mu_F(\lambda u_1 + (1 - \lambda)u_2) \geq \min(\mu_F(u_1), \mu_F(u_2)), \ u_1, u_2 \in U, \lambda \in [0,1]. \ \text{(Convexity)}$$

Definition I.20: *Linguistic Variables*: A linguistic variable is characterized by a quintuple (x,T(x),U,G,M) in which x is the variable's name,T(x) is term set of x, that is, the set of names of linguistic values of x with each being a fuzzy number defined on U; G is a syntactic rule for generating the names of values of x; and M is a semantic rule for associating with each value its meaning. These are explained in the example below.

Example I.7:

Let *x = speed*. This is then interpreted as the *linguistic variable*. Its term set, T(x) could be

T(*speed*) = {*slow,moderate,fast,very slow, more or less fast,...*}.

The members of this set are also called *labels* or *linguistic terms*.

Each of these members is characterized by a fuzzy set in a universe of discourse of speed, say U = [0, 100] (mph). At our own reasonable discretion, we might interpret the following:

Slow = {speed below 40 mph}

Moderate = {speed close to 55 mph}

Fast = {speed above 70 mph}

The *linguistic variable* "speed" with the three terms namely, slow, moderate and fast is represented by the membership function below.

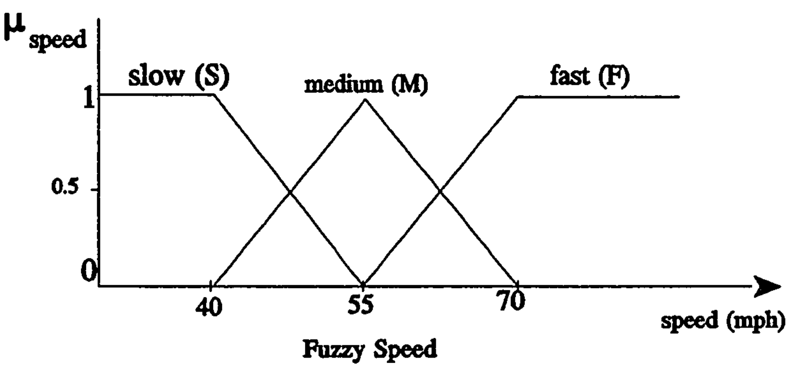


Figure I.4 Membership function of Fuzzy Speed.

The use of fuzzy set provides a basis for a systematic way of handling vague and imprecise concepts. In particular, we can employ fuzzy sets to represent linguistic variables whose "values" are labels or linguistic terms as we have just seen.

I.3 Fuzzy Logic and Approximate Reasoning

Below we explain the two important fuzzy implication inference rules namely, the *generalized modus ponens* (GMP) and the *generalized modus tollens* (GMT). Consider the fuzzy sets A, A′, B, B′ and the linguistic variables x and y:

GMP:

premise 1: x is A′,

premise 2: if x is A then y is B,

consequence: y is B′

GMT:

premise 1: y is B′

premise 2: if x is A then y is B,

consequence: x is A′.

The fuzzy implication inference is based on the compositional rule of inference for approximate reasoning [1]. The GMP, which reduces to 'modus ponens' when A′ = A and B′ = B is closely related to the forward data-driven which is particularly useful in fuzzy logic control. On the other hand, the GMT reduces to 'modus tollens' when B′ = not B and A′ = not A, and is closely related to the backward goal-driven inference which is commonly used in expert systems, especially in diagonizing situations.

Definition I.21: *(Zadeh's) Max-min Compositional Rule of Inference*: if R is a fuzzy relation in U x V, and x is a fuzzy set in U, then this rule of inference asserts that the fuzzy set y in V induced by x is given by $y = x \circ R$ [1], where 'o' is the max-min operator.

Definition I.22: *Conditional Statement*: If A Then B, conventionally denoted by $A \Rightarrow B$. In fuzzy sense, this is denoted by $A \rightarrow B$ (fuzzy implication). Implication here is treated as a cartesian product, that is:

$$A \rightarrow B = \neg A \vee B \triangleq A \times B = \min(\mu_A, \mu_B).$$

Furthermore, consider the rule set below:

R1: If *x* is *A1* and *y* is *B1* then *z* is *C1*

R2: If *x* is *A2* and *y* is *B2* then *z* is *C2*

... ...

Rn: If x is An and y is Bn then z is Cn,

where *x, y, z* are linguistic variables representing two process state variables and one control variable, *z*. Ai, Bi and Ci are linguistic 'values'(terms,labels) of the linguistic variables *x, y, z* in the universes of discourse U, V and W, with i = 1 to n, representing the number of rules.

Definition I.23: *Fuzzy Implication*: is a fuzzy relation Ri and is defined by:

$$\mu_{Ri} \triangleq \mu_{(Ai \text{ and } Bi \rightarrow Ci)}(u,v,w) = [\mu_{Ai}(u) \textit{ and } \mu_{Bi}(v)] \rightarrow \mu_{Ci}(w),$$

where '*Ai and Bi*' is a fuzzy set in U x V. Then:

$Ri \triangleq (Ai \text{ and } Bi) \rightarrow Ci$ is a fuzzy implication (relation) in U x V x W.

Again, to infer the output *z* from the given process states *x, y* and the fuzzy relation R, the sup-star compositional rule of inference (Definition I.21 above) is applied and we have:

$$z = y \circ (x \circ R)$$

Finally, we give below another way to express the implication operation.

Ri: If(*u is Ci* → (*x is Ai and y is Bi*)) then *u is Ci*.

In linguistic terms, the rule is interpreted as "*If the performance index x is Ai and index y is Bi when the control action u is chosen to be Ci*, then this rule is selected and the control action *Ci* is taken to be the output of the controller. This is a commom form of predictive control.

APPENDIX II

FUZZY LOGIC CONTROL PROCESS

II.1 General Fuzzy Logic Control Process

The design process of a fuzzy logic control system can be put into four main categories: 1, Fuzzification 2, Knowledgebase 3, Inferencing and 4, Defuzzification.

1. **Fuzzification**: Briefly, this is the process of making the incoming data fuzzy. Given the available input variables, their ranges are scaled and mapped into manageable universes of discourse. Membership functions and linguistic variables are then assigned.

2. **Knowledgebase**: This usually involves extensive simulations to determine the operating envelope of the particular system. So the result is a knowledge of the application domain and the scope of control magnitudes and objectives. A lot of expert knowledge and heuristics are employed here in order to curtail a possible data explosion. Redundant and nonsensical rules and control objectives are discovered and rejected.

3. **Inferencing**: The heart of fuzzy logic control is the inferencing mechanism. Fuzzy implications (IF-THEN rules) are employed and control actions are inferred based on the rules of inferencing, and the rule of the preponderant alternative. Here, Zadeh's compositional rule of inferencing [2], [4] is used due to finiteness consideration.

4. **Defuzzification**: This is the process of removing the fuzziness from the inferred control action data before applying it to control the process. It involves a scale mapping to convert the range of values of the output variable into the real and actual universes of discourse. The center of gravity method or the peak membership function method is commonly employed. These steps are treated excellently in [65]. A block diagram of a generic fuzzy logic control system is shown in Figure II.1.

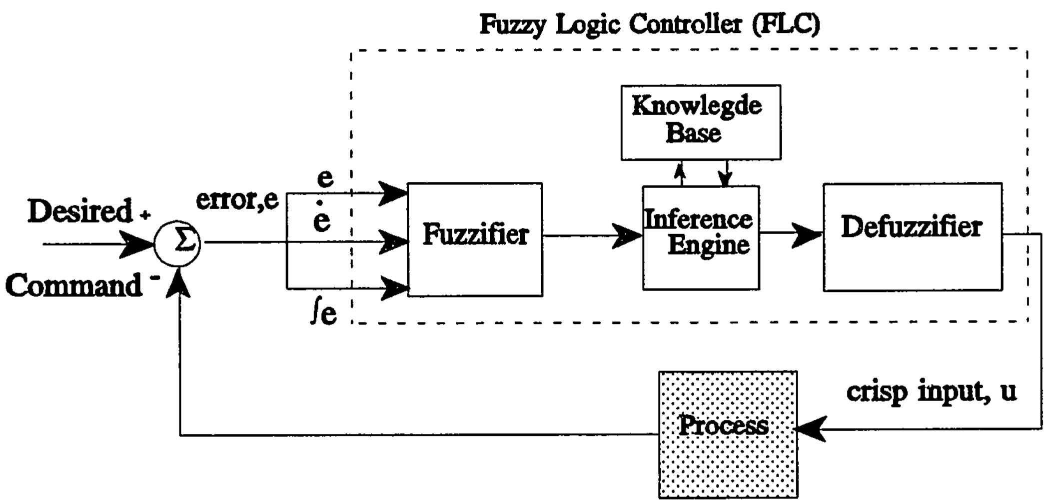


Figure II.1 Generic Fuzzy Logic Control System.

The next section ties up the fuzzy control concepts introduced above into a simplified but typical fuzzy control scheme.

II.2 Fuzzy Control Scheme

As already mentioned, whenever fuzzy sets and fuzzy logic are applied to control, the result is fuzzy control. Fuzzy control was the first application of fuzzy theory to get attention, and it is a field in which research has forged ahead. Some of the application areas include the control of a cement kiln, electric trains, water purification plants, aeroplane, refrigerators, cameras to name a few. This section discusses a scheme for fuzzy logic control, emphasizing the steps already discussed, in particular, the inferencing step. Consider a process, f, to be controlled in the feedback system in Figure II.1. $e_1, e_2, \ldots, e_n$ are the states of the process among which could be error, change in error, integral of error, etc. $u_1, u_2, \ldots, u_m$ are the inputs to the process. y is the observed system output which is compared through a feedback loop with a prescribed set point. Fuzzy control describes the algorithm for process control as a fuzzy relation between information about the condition, $\mathbf{e} \in \mathbf{R}^n$, of the process, $\mathbf{f}$, and the input to the process, $\mathbf{u} \in \mathbf{R}^m$, This control algorithm is given in the form of if-then rules below.

Rule *i*: If e_1^i is *small* and e_2^i is *big*, ..., and e_n^i is *positive medium* then u_1^i is *negative medium* and u_2^i is *positive large*,..., and u_m is *negative medium.*

This is called a fuzzy control rule. The if-part is the *antecedent* and the then-part is the *consequent*. The labels small, big, negative medium, etc., are linguistic terms and are represented as fuzzy sets. The totality of the fuzzy control rules, $\cup R_i$, constitutes the fuzzy control rule base, or the fuzzy controller. The closed-loop fuzzy control system may be represented as:

$$\sum : \left[\bigcup_{i=1}^{s} R_i ; f(x_k, u_k) \right] \qquad \text{(II.1)}$$

where $f(.,.)$ is the model of the process to be controlled with its actual crisp states x_k and crisp inputs, u_k; s is the number of rules. Alternatively, one can now work completely in a fuzzy domain, and the closed-loop fuzzy dynamical system is represented as:

$$\sum : (\mathcal{P};\ \tilde{x}_k, \tilde{u}_k) \qquad \text{(II.2)}$$

where the quantities in tilde are fuzzy sets, and $\mathcal{P}$ is a fuzzy relation formed from the set of fuzzy rules { R_i } as:

$$\mathcal{P} = \bigvee_{i=1}^{s} \{ \tilde{x}^i \wedge \tilde{u}^i \} \triangleq \tilde{x} \circ \tilde{u} \qquad \text{(II.3)}$$

where 'o' is the max-min composition, $\wedge$ and $\vee$ are min and max operations, respectively.

II.2.1 Fuzzy Inferencing

Fuzzy sets or membership functions may be represented in either continuous or discrete form. Figure II.2 below illustrates the two cases. In these illustrations, the actual universe of discourse has been normalized to [-1, 1] and the crisp values of the relevant process variables in this interval are mapped into a grade of membership in [0, 1]. In the discrete case in table 1, the universe of discourse is not normalized. Still, the actual process states $\in$ [-6, 6] are mapped to [0, 1].

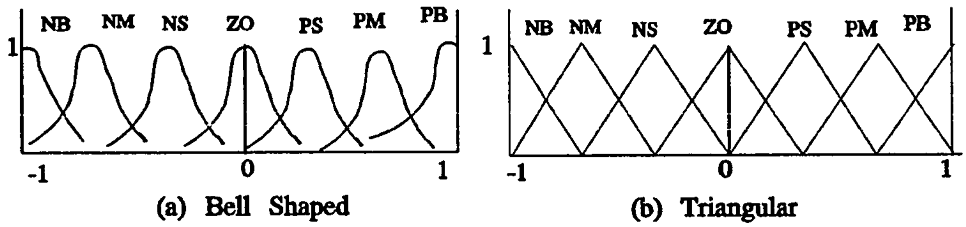


Figure II.2 Continuous Fuzzy Variables

Table II.1 Discrete Fuzzy Variables

	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6
PB	0	0	0	0	0	0	0	0	0	0	.3	.7	1
PM	0	0	0	0	0	0	0	0	.3	.7	1	.7	.3
PS	0	0	0	0	0	0	.3	.7	1	.7	.3	0	0
ZO	0	0	0	0	.3	.7	1	.7	.3	0	0	0	0
NS	0	0	.3	.7	1	.7	.3	0	0	0	0	0	0
NM	.3	.7	1	.7	.3	0	0	0	0	0	0	0	0
NB	1	.7	.3	0	0	0	0	0	0	0	0	0	0

Example II.1

Consider a 2-input, 1-output process with the following 2 rules:

Rule 1. if e_1 is A11 and e_2 is A12 then u is B1

Rule 2. if e_1 is A21 and e_2 is A22 then u is B2

Aij, Bi, i,j = 1,2 are linguistic terms such as small, large, etc., and are represented as fuzzy sets or membership functions. In process set-point control, for example, one may consider $e_1 = e$, the error, and $e_2 = \Delta e$, the change in error. Suppose at some instant of time, these are $e_1 = e_{10}$ and $e_2 = e_{20}$. These are fuzzified by assigning a degree of membership for e_{10} in the fuzzy set A11 as $\mu_{A11}(e_{10})$. Similarly, e_{20} in A12 is fuzzified as $\mu_{A12}(e_{20})$. Next, the degree of fulfillment for these rules is determined as $w_i = \mu_{Ai1}(e_{10} * \mu_{Ai2}(e_{20})$, i = 1,2. Commonly, * is a minimum operator. Using the correlation product approach, the consequent membership functions B1 and B2 are modulated by the w_1 and w_2 respectively. The output fuzzy set is given by $B^* = w_1B1 \cup w_2B2$. The complete inference result u_0 is commonly found in terms of the center of gravity of B^*. That is, B^* is defuzzified to obtain the crisp inferred value of u as $u_0 = \int B^*(u)\, u du \,/ \int B^*(u)\, du$. The entire procedure is depicted in Figure II.3. References [4],[6],[73] and [77] provide excellent introductions to fuzzy logic control.

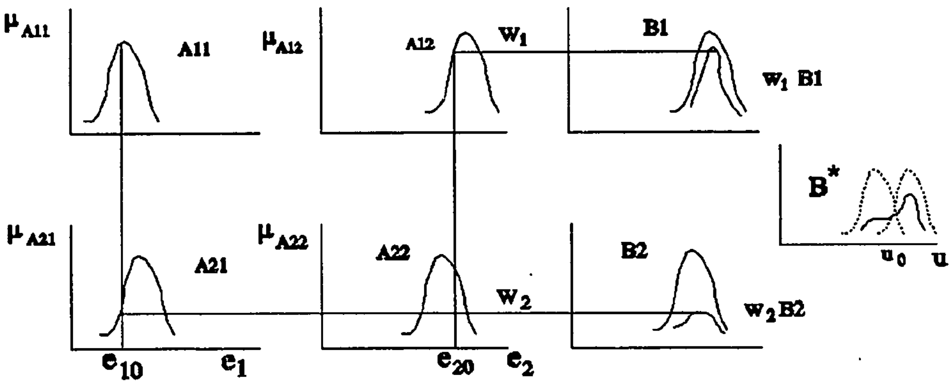


Figure II.3 An Inference Method

II.2.2 The Weighted Average Inferencing

This gives a quick estimate of the crisp, representative action to be taken at the consequent side of the rule. Given the degrees of fulfillment w_1 for rule 1 and w_2 for rule 2, the weighted average inferencing is determined as

$$u_0 = \frac{w_1 \mu_{B1}^{-1}(w_1) + w_2 \mu_{B2}^{-1}(w_2)}{w_1 + w_2} = \frac{w_1 u_{10} + w_2 u_{20}}{w_1 + w_2} \tag{II.4}$$

where u_{i0} is such that $\mu_{Bi}(u_{i0}) = w_i$, $i = 1,2$, at the first intersection of w_i with μ_{Bi}.

It is straight forward to show that a fuzzy controller employing the weighted average inferencing method to control a linear plant gives rise to a nonlinear control system. It suffices to show that this inferencing method is generally not linear with respect to the degree of fulfillment, and therefore with respect to the incoming crisp inputs. This can be done by applying the principle of superposition and homogeneity. Both fail to hold, but it is easier to show that homogeneity fails for the general case as is done below.

(proof): Consider the weighted average, WA(u_{i0}), of representative control at the consequent, Bi (or μ_{Bi}). Let α_i be the degree of fulfillment for the antecedents of the i[th] rule. By definition, the weighted average operator is

$$WA(u_{i0};\alpha_i) = \frac{\alpha_1 u_{10} + \alpha_2 u_{20} + \dots + \alpha_s u_{s0}}{\alpha_1 + \dots + \alpha_s} \triangleq u_0 \qquad \text{(II.5)}$$

where s is the number of rules.

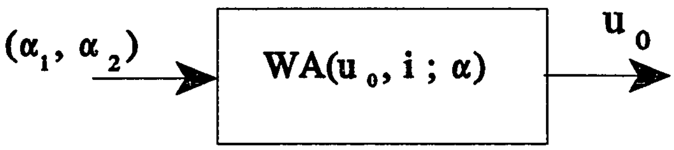


Figure II.4 The Weighted Average Operator

For $\lambda \in \mathbf{R}$, let us form

$$\lambda u_0 = \lambda WA(u_{i0};\alpha_i) = \lambda \frac{\alpha_1 u_{10} + \alpha_2 u_{20} + \dots + \alpha_s u_{s0}}{\alpha_1 + \dots + \alpha_s} \qquad \text{(II.6)}$$

Let us pass $\lambda.(\alpha_1,\dots,\alpha_s)$ through WA(.,.) as in

$$\begin{aligned} WA(u_{i0};\lambda\alpha_i) &= \frac{\lambda\alpha_1 u_{10} + \lambda\alpha_2 u_{20} + \dots + \lambda\alpha_s u_{s0}}{\lambda\alpha_1 + \dots + \lambda\alpha_s} \\ &= WA(u_{i0};\alpha_i) \\ &\neq \lambda WA(u_{i0};\alpha_i) \ or\ \lambda u_0,\ \ for\ \lambda \neq 1. \end{aligned} \qquad \text{(II.7)}$$

Therefore the weighted average inferencing scheme is not linear. ■

APPENDIX III

RELATIONAL FUZZY FEEDBACK SYSTEM REPRESENTATION

III.1 Open Loop System

Consider the open loop system below.

Figure III.1 Open-Loop Fuzzy System.

Let the input u and output x be fuzzy sets defined on U x X repectively. It is assumed that the dynamic behavior of the process is governed by:

$$x_{t+1} = x_t \, u_t \, o \, P \qquad \text{(III.1)}$$

where x_t is the current state, at integer time t, and x_{t+1} is the next state. $x_t \, u_t$ is a fuzzy set on the cartesian product space X x U, and whose membership function is:

$\mu_{x_t u_t}(x,u) = \mu_{x_t}(x) \wedge \mu_{u_t}(u)$, where $\wedge \triangleq$ min operator.

The discrete relation P is defined on X x U x X and represents the next state mapping. Using the max-min composition 'o', we have:

$$\mu_{xt+1}(x_{t+1}) = \vee_{xt\ ut}[\ \mu_{xt\ ut}(x_t,u_t) \wedge \mu_p(x_t,u_t,u_{t+1})\ ].$$

The evolution of the system is studied by employing two important equalities.

Equality 1: If x is a finite discrete set on X and if Q is a finite discrete binary relation on X x X then

$$(x \circ Q) \circ Q = x \circ (Q \circ Q) \qquad \text{(III.2)}$$

Equality 2: if x and u are fuzzy sets on X and U respectively, and if P is a ternary relation on X x U x X then

$$x\, u \circ P = x \circ (u \circ P) = u \circ (x \circ P).$$

The proofs of these are found in [Tong, 1980].

let x_t at t = 0 de denoted by x_o, the initial state, and let $u_t = u_c$, be a constant for all time. Then we can see that:

$$x_1 = x_o\, u_c \circ P \qquad \text{(III.3)}$$

$$x_2 = x_1\, u_c \circ P \qquad \text{(III.4)}$$

Continuing this on N times we have:

$$x_N = x_{N-1}\, u_c \circ P \tag{III.5}$$

If we substitute (III.3) into (III.4) we have $x_2 = (x_o\, u_c \circ P) \circ P$, and upon applying equality 2., we have:

$$x_2 = (x_o\, u_c \circ P) \circ (u_c \circ P) = (x_o \circ (u_c \circ P)) \circ (u_c \circ P) \tag{III.6}$$

Let us define

$$Q = u_c \circ P, \tag{III.7}$$

a square relation on $X \times X$, then $x_2 = (x_o \circ Q) \circ Q$

Upon applying equation (1), $x_2 = x_o \circ (Q \circ Q) = Q^2$. Similarly we express

$$x_3 = (x_o \circ Q^2) u_c \circ P = (x_o \circ Q^2) \circ Q = x_o \circ Q^3.$$

By induction,

$$x_N = x_o \circ Q^N \tag{III.8}$$

Clearly, the asymptotic behavior is dictated by the behavior of Q^N *as* $N \rightarrow \infty$. The pertinent information is given by the solution of the following equation.

$$S_{t+1} = S_t \circ Q \tag{III.9}$$

where S_t is a relation on $X \times X$ with the initial condition $S_o = I$, the identity relation such that $\mu_I(x_1, x_2) = 1$, $x_1 = x_2$ and 0 otherwise.

The solution S_t was found to either converge or cycle with a finite period [Thomason]. Observe that when the input is 'zero' in (III.7), then $Q = P$ and the solution corresponds to the unforced open loop system behavior. The convergence properties of the system are seen to depend only on Q. The actual converged value of x will ofcourse depend on x_o. However, the *existence* of such a state is independent of x_o.

III.2 Closed loop Fuzzy System

Consider the block diagram of the fuzzy feedback system below.

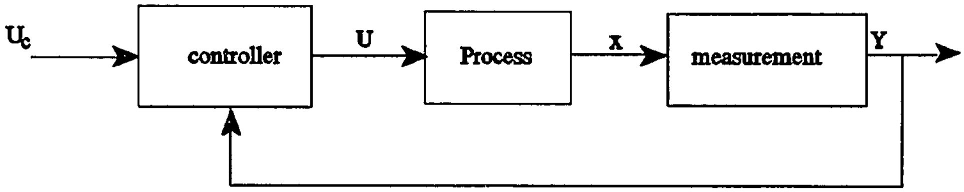


Figure III.2 Fuzzy Feedback System

It is still assumed that the system is governed by the discrete-time equation (III.1) which is stated again below:

$$x_{t+1} = x_t\, u_t\, o\, P$$

Since we are feeding back the output, we have a measurement device with characteristics H such that

$$y_t = x_t \, o \, H \qquad \text{(III.10)}$$

where y_t is a fuzzy set defined on the finite discrete multidimensional space Y. We assume that the controller has no dynamics (memoryless) so that this then compares with the use of simple gain in classical control systems. So let the controller be governed by the discrete-time equation:

$$u_t = y_t \, u_c \, o \, K, \qquad \text{(III.11)}$$

where K is the controller relation defined on $Y \times U_c \times U$. The closed-loop behavior is determined by:

$$x_{t+1} = x_t (y_t u_c \, o \, K) \, o \, P = (y_t u_c \, o \, K) \, o \, (x_t \, o \, P) = ((x_t \, o \, H) \, o \, (u_c \, o \, K)) \, o \, (x_t \, o \, P). \qquad \text{(III.12)}$$

Let us define the constant

$$C = H \, o \, (u_c \, o \, K), \qquad \text{(III.13)}$$

which is a relation on $X \times U$. Then we use this in (III.12) to find that

$$x_{t+1} = (x_t \, o \, C) \, o \, (x_t \, o \, P) \qquad \text{(III.14)}$$

Theorem III.1 : A close loop system described by (III.14) has a discrete-time response given by

$$x_{t+1} = (x_o \; o \; (C \; o \; A_t)) \; o \; ((x_o \; o \; B_t) \; o \; P, \qquad \text{(III.15)}$$

where A_t is a relation on U x U, and B_t is a relation on X x X. These may be computed recursively as:

$$A_{t+1} = A_t \; o \; ((x_o \; o \; B_t) \; o \; P) \; o \; C, \qquad \text{(III.16)}$$

$$B_{t+1} = C \; o \; A_t \; o \; ((x_o \; o \; B_t) \; o \; P, \qquad \text{(III.17)}$$

with initial conditions,

$$A_o = I_U \; and \; B_o = I_X. \qquad \text{(III.18)}$$

We can think of A_t and B_t as a controller output modifier and an initial state modifier respectively. We combine (III.16) and (III.17) to obtain the more preferrable formula for the system behavior as:

$$x_{t+1} = (x_o \; o \; (C \; o \; W_t \; o \;)) \; o \; ((x_o \; o \; C_o \; W_t) \; o \; P), \text{ with} \qquad \text{(III.19)}$$

$$W_{t+1} = W_t \; o \; C \; o \; (x_o \; o \; C \; o \; W_t) \; o \; P, \qquad \text{(III.20)}$$

and initial conditions $W_o \; o \; C = I_U$, $C \; o \; W_c = I_X$. The formula (III.19) is preferred since it enables us to put some meaning into A and B. A complete determination of the closed system behavior is enhanced by Theorem III.2 below.

Theorem III.2: A closed loop system described by (III.19) has a discrete-time response govered by:

$$x_{t+1} = x_o \; o \; S_t, \qquad \text{(III.21)}$$

where S_t is a relation on X x X and may be computed from

$$S_{t+1} = (S_t \; o \; C) \; o \; ((x_o \; o \; S_t) \; o \; P, \qquad \text{(III.22)}$$

with initial condition $S_o = C \; o \; (x_o \; o \; P)$.

Proofs of these theorem are given in [17]. Equation (III.22) plays the same role that (III.17) plays in the analysis of open loop system. However, now the solution depends on both x_o and u_c. Unfortunately, no analytical solution is available. Experiments do show that it also either converges or cycles with finite period, with the rate of convergence and period being dependent upon the cardinality of S.

III.2.1 Control Application

Suppose we write (III.14) as

$$x_{t+1} = x_t(x_t \; u_c) \; o \; ((H \; o \; K) \; o \; P = x_t(x_t \; u_c) \; o \; (G \; o \; P) \qquad \text{(III.23)}$$

where G is a relation on X x X x U_c x X. The problem is to find a K such that

$$x_{t+1} = x_t(x_t \; u_c) \; o \; T, \qquad \text{(III.24)}$$

where T is the desired transition relation, and K is the controller relation. So the control problem reduces to finding the solution of two relational equations. First finding G given P and T, then finding K given H and G. That is,

$$G \, o \, H = T \text{ and} \quad \text{(III.25a)}$$

$$H \, o \, K = G. \quad \text{(III.25b)}$$

If we have complete state measurement, then H = I, and the problem reduces to finding the solution to $K \, o \, P = T$.

III.2.2 Comments

This is an innovative and feasible approach for the description of fuzzy dynamical systems, and an appealing tool for the analysis and design of controllers for complex systems. Some problems do exist however.

1. There is the difficulty of ensuring that the discrete process relation P is a good approximation of the nonfuzzy behavior of the real process. This could be verified through simulations.

2. Since the specification of the transition relation T is done linguistically, it is possible that this may not be unique, thus making the solution of (III.25) a nontrivial task.

3. The analysis can become extremely tedious especially for high cardinality systems where we may have to deal with quite large relational matrices. This computational burden can be alleviated by devising an efficient automated scheme for evaluating the various relations.

APPENDIX IV

A SYSTEMATIC DESIGN METHODOLOGY

IV.1 Introduction

This appendix introduces a novel systematic fuzzy logic control design methodology and applies it to control the idle speed of an automotive engine. A method is proposed to design fuzzy logic controllers for a class of nonlinear systems using the direct intelligent control paradigm. The procedure is based on a partitioning of the state space into small rectangles called cell-groups, and quantization of the states and the available controls into finite levels or bins. Membership functions are then assigned for the state and controls. The transition from one conditional subspace to another is accomplished via a center-point mapping of the cell-groups, under the applied action of each if-then rule. The systematic aspects of the procedure refer to (a) the way transitional elements are clustered on the basis of an explicitly defined performance measure; (b) optimum transitions are selected using a search procedure; (c) finally, the fuzzy rules are generated automatically for transitioning from any initial cell-group to the target cell-group. A simplified engine model for idle speed control is used as the testbed for development and demonstration purposes.

IV.1.1 Background

The control of complex nonlinear systems has been approached in recent years using fuzzy logic techniques in an expert system setting. Since the pioneering work of Mamdani and his co-workers [12], most fuzzy logic control routines have relied upon heuristic information (based on operators' experience) to design the knowledge (rule) base. For an excellent survey of fuzzy logic control methods, the reader is referred to [4]. The lack of formalism, in the control-theoretic sense, has limited severely the introduction of such conventional performance metrics as stability, robustness and optimality, especially when the control problem is viewed as an expert system paradigm. Nevertheless, fuzzy logic control has found numerous applications in such diverse areas as consumer electronics, motion control, industrial processes, etc. The control of an automobile's idle speed has been addressed using various conventional control schemes [48],[45] and others, or fuzzy logic methods as in [47]. Most of the these rely primarily upon heuristics to derive the rule base and, therefore, lack the formalism of a more "generic" design approach. This paper is attempting to address these concerns by bridging the gap between control-theoretic concepts and artificial intelligence based tools.

The first section presents a two-stage, nonlinear engine dynamic model under idle speed conditions. The fuzzy logic control problem is introduced next and the automatic rule generation procedure is detailed. The fuzzy controller structure is then described. Some simulation results are then presented. Finally, we comment on the results.

IV.2 Engine Model

The class of systems we are considering is generally represented by:

$$\dot{x} = f(x,u,t,d), \quad x \in \mathbf{R}^n,\ u \in \mathbf{R}^m,\ t \in R^+, d \in \mathbf{R}, f \in \mathbf{R}^n \tag{IV.1}$$

where **x** are the states, **u** the control inputs, d is a scalar disturbance term, and f is a nonlinear mapping. A two-state engine model belongs to this class and is given by [48],[45]:

$$\begin{aligned} \dot{P} &= k_p(\dot{m}_{ai} - \dot{m}_{ao}) \\ \dot{N} &= k_N(T_i - T_L) \end{aligned} \tag{IV.2}$$

Where, more explicitly:

$$\begin{aligned} \dot{m}_{ai} &= (1 + 0.907\theta + 0.0998\theta^2)\, g(P) \\ \dot{m}_{ao} &= -0.0005968N - 0.1336P + 0.0005341NP + 0.000001757NP^2 \\ T_i &= -39.22 + \frac{325024}{120N}\dot{m}_{ao} - 0.0112\delta^2 + 0.000675\delta N(2\pi/60) \\ &\quad + 0.635\delta + 0.0216N(2\pi/60) - 0.000102N^2(2\pi/60)^2 \\ T_L &= (N/263.17)^2 + T_d \\ g(P) &= \begin{cases} 1 & P < 50.66 \\ 0.0197(101.325P - P^2)^{\frac{1}{2}} & P \geq 50.66 \end{cases} \end{aligned} \tag{IV.3}$$

where P is the manifold pressure in KPa and N is engine speed in RPM. (see Figure IV.1) and [46].

δ Spark advance (between 10 and 45 degrees)

θ Throttle angle (between 5 and 35 degrees)

T_d Accessory load between 0 and 61 Nm.

$\dot{m}_{ai}$ Mass flow rate into the manifold.

$\dot{m}_{ao}$ Mass flow rate out of the manifold and into the cylinder.

T_i Internally developed torque in (Nm).

T_L Load Torque in (Nm) - made up of accessory torque T_d and shaft torque.

$g(P)$ Manifold pressure influence function.

k_p Manifold dynamics constant.

k_N Rotational dynamics constant.

These equations are highly nonlinear. k_P and k_N are empirically determined and are not know accurately. The dynamic equations themselves are derived form static maps and as such entail a great deal of uncertainty.

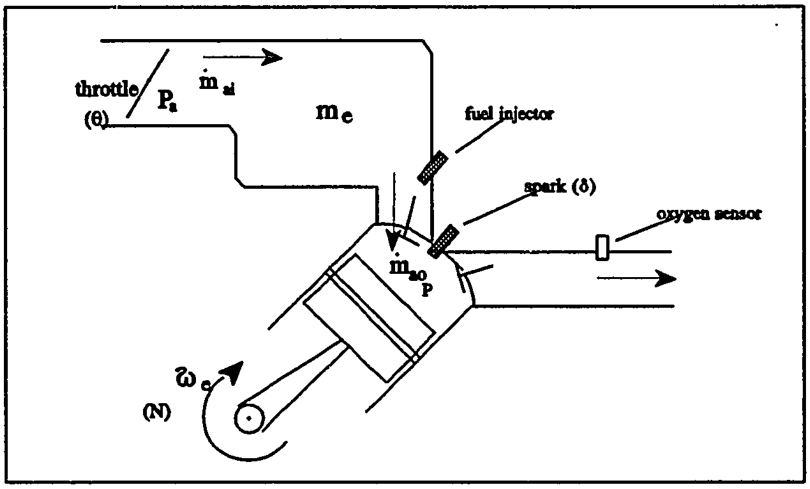


Figure IV.1 The Main Engine Sub-system

The dynamic equations (IV.2) can be transformed into the form (IV.1) where $f \in \mathbb{R}^2$, $d = T_d$. We define the state vector $x \in \mathbb{R}2$, and the control vector $u \in \mathbb{R}2$, where

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \triangleq \begin{pmatrix} P \\ N \end{pmatrix}, \quad u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \triangleq \begin{pmatrix} \theta \\ \delta \end{pmatrix} \tag{IV.4}$$

The nominal (equilibrium) operating point is typically determined on the basis of operating conditions, minimum energy dissipation, etc. (in the test engine considered it is set at $N = N_0 = 750$ rpm at a corresponding equilibrium pressure, $P = P_0 = 34.25$ kPa). The speed and pressure errors are defined respectively as, $e_N = N_0 - N$, and $e_P = P_0 - P$.

IV.3 Problem Statement

Our primary fuzzy logic control objective for the complex nonlinear engine model is to maintain the speed trajectory, x_2, to within a specified tolerance of the equilibrium value of 750 rpm. But in the presence of an external disturbance, such as the onset of the air conditioning unit, the equilibrating speed is raised by about 25 rpm per load, each load being about 20 Nm up to a maximum of 60 Nm. So, depending on prevailing load conditions, the operating or idling speed is really not fixed. Thus, we require that the fuzzy controller transition smoothly to various idle speeds all of which are in a 60 Nm-neighborhood of the no-load idle speed of 750 rpm.

Nonlinear fuzzy regulators can be used to control a system with fixed operating conditions; however, this approach is restrictive in that a large number of knowledge bases may be required corresponding to different operating conditions. Therefore, a fine tracking controller is needed to resolve such complex control situations. A framework for fuzzy control theory is established herein to address the systematic control rule base design problem.

IV.4 Fuzzy Logic Control

Consider the fuzzy logic controller design for a nonlinear process of the class of (1) described simply by

$$\dot{x} = f(x(t), u(t)) \tag{IV.5}$$

Let the desired profile be x^d. We define the state error as $e = x^d - x$.

As a first step, we represent the resultant control mode, from fuzzy inferencing, functionally by:

$$u(t) \doteq g(e(t), x^d(t)) \tag{IV.6}$$

The error system is given by

$$\dot{e} = \dot{x} = f((x^d - e), g(e, x^d)) \triangleq h(e, x^d, \dot{x}^d) \tag{IV.7}$$

The objective of the fuzzy controller is to maintain closed-loop stability of the error system, i.e. make e(t) converge to zero. The regulation is performed via a fuzzy hypercube [50], using the phase portrait assignment algorithm [49]; with the control law given as:

$$u = g(e, x^d) = g(\int h d\tau, \dot{x}^d) \tag{IV.8}$$

The construction of the fuzzy rule base depends on the phase portrait of the state x(t), the error e(t), the desired profile, x^d, and the quantization specifications. A detailed description of the phase portrait assignment algorithm and the fuzzy hypercube as its implementation platform may be found in [49] and [50].

IV.4.1 Automatic Rule Generation

A step-by-step procedure for synthesizing a fuzzy logic rule base on the basis of data obtained from numerical simulations is summarized below. The engine model is used as the test platform to illustrate the procedural steps. It is assumed first that the system is known but ill-defined because of the inherent uncertainty associated with the model. From the design standpoint, an accurate mathematical representation of the system is not required, but an approximate or simplified model is acceptable.

A finite set of representative center points in the state space is chosen to anticipate the trajectories from one subspace to another. Each state variable ranges a permissible set of permissible values. N, for example, can take values in [350,1150] RPM. Now, each interval associated with a state variable, is quantized into smaller subintervals. The number of the subintervals for each state variable is chosen based on: (i) the specified ranges of each variable and the behavior of the system in terms of accuracy and resolution, (ii) the number of rules that are considered manageable in the sense of minimizing the heuristic search time and reducing computational complexity. The latter is accounted for in terms of memory storage and computational speed. We denote by E_{1m} and E_{2m} the m-th and n-th subintervals of the x_1 and x_2 range, respectively. Linguistically, we define

$$L_{mn} = E_{1m} \times E_{2n}$$

and then L_{mn} is a finite region in the state space. Figure IV.2 shows the space partitioning and cell-group numbering employed in the scheme. The term "cell-group" is adopted to indicate that a partition is either a single cell or a grouping of cells resulting from the application of an appropriate clustering algorithm.

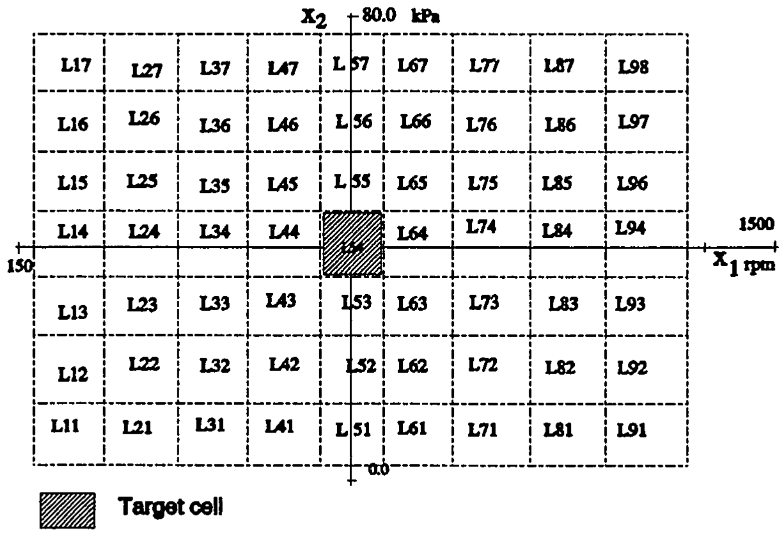


Figure IV.2 The Cell-group Space (Target Cell: 750 rpm, P=34 kPa, Td = 0)

Now consider the space of the control variables. A range of permissible values is associated with each control variable. Quantize next each of these intervals into N_{u1}, N_{u2} equal subintervals, for u_1 and u_2 respectively, to derive $N_{u1}+1$ intermediate values for u_1, and $N_{u2}+1$ intermediate values for u_2. Again, the numbers N_{u1} and N_{u2} are selected according to criteria similar to those for the states. The behavior of the functions dictate whether a coarse or fine quantization is used. This scheme results in a set S1 of values θ_1, θ_2, .., θ_{Nu1+1} for the control variable u_1, and another set S_2 of values δ_1, δ_2, .., δ_{Nu2+1} for u_2. The idea now is the following: for each cell-group, consider the center point. This point is the initial state for the simulation. Apply to the system the constant input (θ_i,δ_j), for all (θ_i,δ_j) in the set $S=S_1 \times S_2$, and perform a siimulation run until the system enters another cell-group in the state space or until the simulation time becomes larger than a fixed value t_o (t_o= 1 sec was used). So, for each cell-group in the state space, we perform ($N_{u1}+1$)($N_{u2}+1$) simulations, which result in ($N_{u1}+1$)($N_{u2}+1$) transitional relations of the type:

$$(\theta_1,\delta_1) : L_{11} \rightarrow L_{12}, \qquad (\theta_1,\delta_2) : L_{11} \rightarrow L_{21},\ldots,(\theta_3,\delta_1) : L_{11} \rightarrow L_{11}.$$

The first transitional relation relays the following information: if the system starts at the center of the cell-group L_{11} and the constant input (θ_1,δ_1) is applied, then the system will end up in the cell-group L_{12}. The time required for the transition is stored, as well as the energy of the control law (u^Tu) and the Euclidean distance of the state from the equilibrium point, at the end of each simulation. Note that the terminating cell-group can either be the same as the starting cell-group, in which case it is called an 'invariant manifold', or an adjacent one. In other words, the sampling period of the simulation is small enough so that no long jumps between cell-groups are allowed. This is a rather stringent condition which is hard to satisfy in general. However, it may be satisfied for well-behaved members of (1), and also under a uniform cell-size construction. Still, some ad hoc adjustment of the sampling time may be needed. From the above, it is clear that the target cell-group L_{mn} (the specified goal) must be an invariant manifold for some (δ_i, θ_j)-pair. This is also necessary for convergence and asymptotic stability. This is equivalent to the 'reachability condition' in modern control theory. Now, after all pairs of control values in S have been used, continue with the next cell-group. Repeat until all cell-groups in the state space have been simulated.

At the end of this phase, the following information is stored: if the system starts at cell group L_{ij} and the constant control values θ_k,δ_l are applied, then the system will enter cell-group L_{mn} in a fixed time (counted in msec.), having wasted a certain amount of energy and being at some distance from the equilibrium. This is done for all cell-groups and for all control values in S.

The next step is to search this data file and come up with the following result: if the system starts in cell-group L_{ij}, then in order to drive it eventually to the equilibrium point, the 'best' control to be applied is θ_k,δ_l; and such a rule has to be derived for all cell-groups in the state space. 'Best' can be in the sense of minimum time or minimum energy or minimum error

$$J = [\alpha(e^Te) + \beta(u^Tu) + \tau]t \qquad \text{(IV.9)}$$

(distance from the equilibrium).The general form of the cost of each transition is given as where u is the control input applied while in L_{mn} in order to transition to the next cell-group. α, β and τ are binary coefficients for selecting a criterion as squared error, energy or time, respectively. For example, for minimum time control we set $(\alpha,\beta,\tau)=(0,0,1)$. The this search is carried out is described next. Each cell-group becomes a node of a graph. Each transition becomes a unidirectional arc with weight equal to the transition's cost, in the sense defined above. It should also be clear that each transition (i.e. arc) is associated with a pair of constant control values. The purpose is to find a path from every node to the equilibrium node, with the smallest possible sum of arc weights. Before the search procedure is given, let us clarify some points. The unidirectional arcs are allowed to form loops between two or more nodes. This is so because, if there is a path from any of the nodes in the loop towards the equilibrium node, the algorithm is guaranteed to find it, i.e. it will not run for ever because of the loop. If there is more than one arc that starts at node A and ends at node B, then all but the one with the smallest weight can be eliminated. This is so because if a transition has to be made from node A to node B, the transition with the smallest weight will be chosen.

IV.4.2 The Search Procedure

Define the main list as the list that contains the nodes for which we know what control to apply in order to drive the system eventually to the equilibrium node. Initially the main list is empty. Define the nonbasic list as the list that contains all nodes from which there is an arc towards any node in the main list. This is initially empty as well. At the end of the search procedure, the main list contains all nodes for which there exists a path that connects them with the equilibrium node, and the nonbasic list is empty.

The search technique proceeds along the following steps:

Step 1 : Enter the node corresponding to the equilibrium cell-group into the main list.

Step 2 : Expand the last node entered into the main list, i.e. find all arcs that end up in the node that was entered last into the main list. Then enter the starting node of each one of these arcs provided that it is not already in the main list, into the nonbasic list. If a node is already in the nonbasic list, then there already exists a path from this node to the equilibrium node. If the new path has less total cost (from this node to the equilibrium node) than the existing one, then update the total cost associated with that node.

Step 3 : If the nonbasic list is not empty, then find the node in the nonbasic list with the smallest cost, enter it into the main list, and remove it from the nonbasic list, else stop.

Step 4 : Go to Step 2.

The algorithm is guaranteed to terminate in a finite number of steps and to find the best path from any node to the equilibrium node, provided that such a path exists. At the end, the graph is a tree and there exists a unique path from every node to the equilibrium node. As we already mentioned, each arc is associated with a pair of constant control values. So, each arc of a path that connects any node with the equilibrium node, corresponds to a pair of control values. Basically, this information suggests the following strategy: if the system starts in cell-group L_{mn}, apply the control corresponding to the first arc of the path connecting L_{mn} with the equilibrium cell-group, until the system enters another cell-group. Then the new path is the previous one after the first arc is removed. The control associated with the new first arc is applied to the system, until we enter another cell-group. This is repeated until the system has reached the equilibrium cell-group.

IV.4.3 Fuzzy Controller Structure

Once the search procedure generates the optimal tree, the fuzzy controller design may be addressed next. Some points need to be clarified here. Every single transition that is part of this tree, will become a rule in the fuzzy controller. Since each transition of this tree corresponds to a node (i.e. cell-group), we can equivalently state that the number of rules is equal to the number of cell-groups from which there is a path towards the equilibrium cell-group. The procedure may be summarized as follows: if the system starts in cell-group L_{mn}, apply the control θ_k, δ_l for all cell-groups in the state space. The way the problem is set up by now, a point in the state space can belong to only one cell-group. The control values are crisp as well. The next step is to fuzzify the borders of the cell-groups and the control values. One way to fuzzify the borders between the cell-groups is to introduce a linguistic term (membership function) for each dimension of a cell-group (see Figure IV.6a). By doing this, a point in the state space does not have to belong to only one cell-group, but it can belong to many cell-groups with different degrees of membership in each one. This means that many rules may be fired at the same time. This is good because it gives smooth transitions of the actual system. On the other hand, fuzzification of the control space can be obtained by introducing a linguistic term (membership function) for each value in the sets S_1 and S_2.

The center of each membership function is located at the point θ_k or δ_l. There is overlapping between the membership functions because this makes the control law a continuous function of time.

The states, along with the control variables, define the premises and consequents of the fuzzy rule base. So, we are proposing a two-premise and two-consequent structure. The general form of the rules is given below:

$$\text{If } (N \text{ is } F_1) \ \& \ (P \text{ is } F_2) \ \text{ THEN } \ (\delta \text{ is } G_1) \ \& \ (\theta \text{ is } G_2).$$

where F_i and G_i are the premise and consequent linguistic terms, respectively. We denote the control and the corresponding rule should be fired with degree of fulfillment equal to the minimum of the two degrees of membership. resulting from this as u_{FLC}. Note that the fuzzy controller operates on N and P, as shown in Figure IV.3.

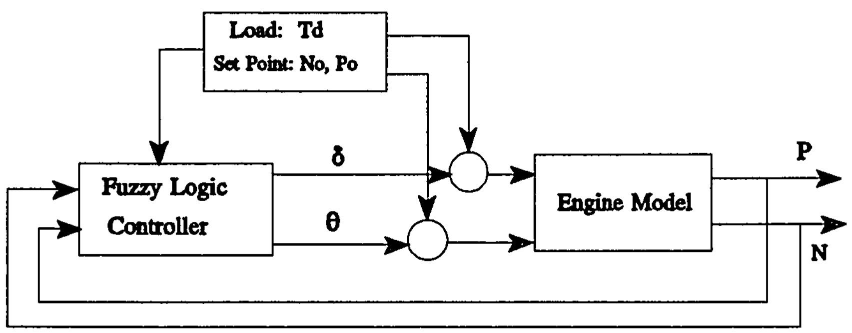


Figure IV.3 The Fuzzy Idle Speed Controller main Control Loop

When an accessory load is activated, we no longer equilibrate at the nominal values N_0 and P_0. We specify a higher idling speed and solve equation (IV.5) for a new u (with $T_d \neq 0$) as, $f(x_e,u,T_d) = 0$, where x_e is the new equilibrium point. So, the accessory load, T_d, enters into a higher level controller in the form

$$\text{If } T_d \in (0, 61] \text{ Nm then } u = u_{FLC} \pm \text{constant value.}$$

Now, each one of the two expressions (N is F_1) and (P is F_2) is true with a certain degree of membership This degree of fulfillment determines the effect of this rule on the calculation of the final output of the fuzzy controller. The consequence part of each rule is scaled according to the degree of fulfillment. The contribution from all the rules is taken into consideration, in order to come up with a fuzzy representation of the control variables. Then, by using any standard defuzzification method, the crisp values of the control variables are computed and are fed into the system.

A center-point mapping is employed for the purpose of simplifying the computations. That is, all initial states in a given cell-group are replaced by the center-point of the cell-group. We could have chosen to explore more points in each cell-group and then assign a mean value to the corresponding initial state. Our simplification, though, does not compromise the uniqueness of the solution of the system given in (IV.1). Details of the center-point mapping technique may be found in Hsu [31].

Finally, we extract only one transition based on the optimal strategy. The elements of the finalized tree from the search, constitute a control rule base which is automatically generated by the computerized design procedure. Simply, the transitional relations that force the trajectories from any initial points in the state space to the desired goal, within the prescribed tolerances, are themselves the control rules for feedback regulation. The trade-offs between the number of quantization levels in L_{mn} and the smoothness in transition, or the total numbers of E_{1m}, E_{2n} and the controller performance are resolved via heuristics. The evolution of the cell-groups in the state space is similar to the work of Chen [5].

IV.5 Simulation Studies

The engine model has been tested separately under various operating conditions and was found to be in good agreement with experimental results provided by Ford Motor Company. Figure IV.4 depicts the phase portrait of the cell-groups selected by the automatic rule generation algorithm and the associated search procedure. The rule for each cell-group is automatically placed in the cell-group. For instance, a cell-group may be designated as "L54 L32" on it. This is read as:

If *P and N* are in Cell *L54*, Then control is *U32*.

where U32 means the *3rd* quantized value for theta and the *2nd* quantized value for delta. Optimum results are obtained when the rule base is composed of 56 rules. The simulation results show such desirable properties as stability and robustness with respect to small accessory load perturbations. A rigorous treatment of the stability analysis of the fuzzy controller is addressed in [56].

Figure IV.5 depicts the phase plane trajectory with some arbitrary initial conditions. The fuzzy control law minimizes the squared error in this case. The same figure also shows the time response for this no load situation for the two key engine variables, namely, shaft speed and manifold pressure. Figure IV.6(a) depicts the phase portrait with minimun time(A) control. Figure IV.6(b) compares the phase trajectories for minimum time(A), minimum energy(B) and minimun squared error(C). Figure IV.6(c) (top), shows the corresponding time plots for the shaft speed and manifold pressure, respectively. Figure IV.6(c)(bottom) shows the control action required for δ and θ under the various strategies. Finally, Figure IV.7 extends these results in the presence of a large engine disturbance. A 20-Nm disturbance is applied to the engine simulating an air conditioning load. The load is applied after the engine has reached the equilibrium condition. The magnitude of the disturbance and the limited available control authority dictate a new equlibrium setting. The engine equilibrates now at about 776.50 rpm and at a higher manifold pressure. The phase plane trajectory and the corresponding time plots are depicted in Figure IV.7. It is noted that the speed response, under minimum squared error conditions, is smoother and the performance of the fuzzy logic closed-loop controller satisfactory. Comparisons were made also with a fixed controller of the state feedback variety. The state feedback controller performed comparably well but with a slower response time. Its performance degenerated significantly in the presence of disturbances. A significant improvement is, therefore, achieved via the rule base controller.

IV.5.1 Simulation Results

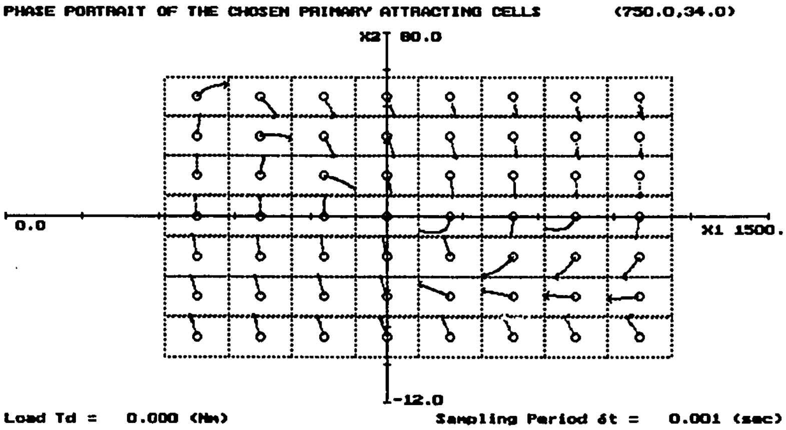


Figure IV.4 The Phase Portrait in the State Space

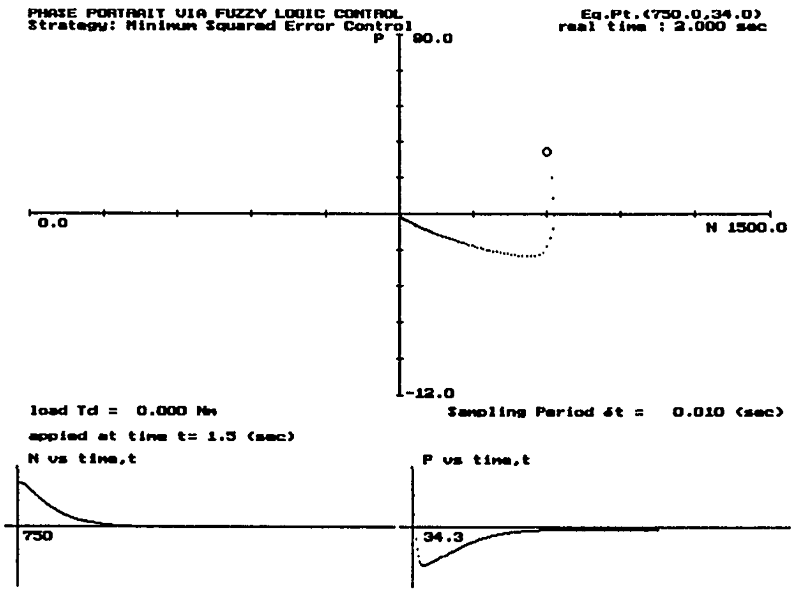


Figure IV.5 Phase Portrait and Time Response characteristics (no-load)

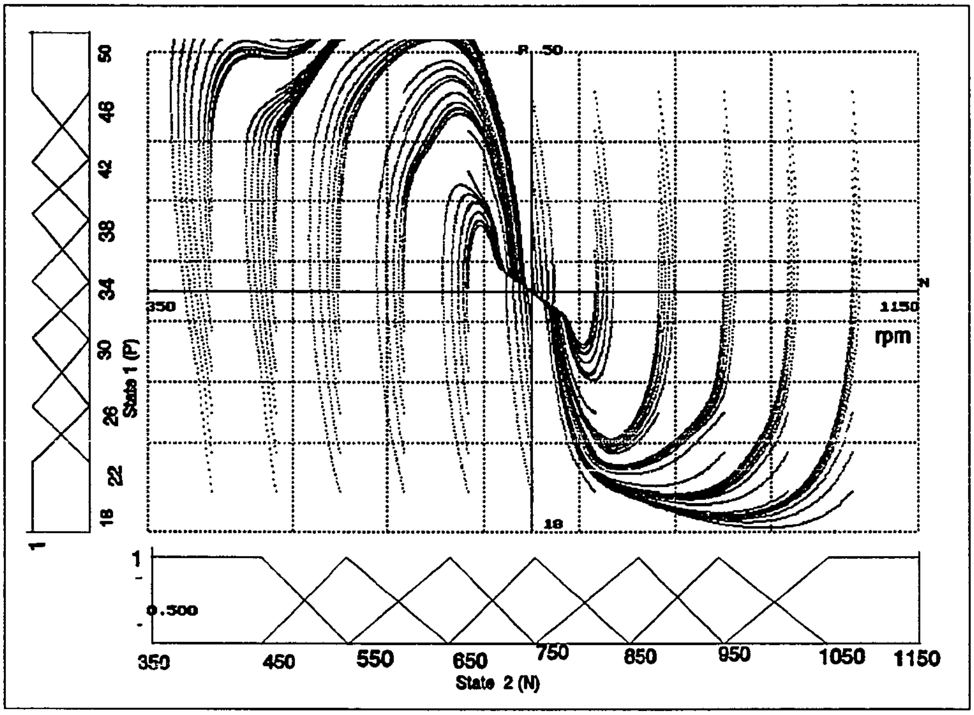


Figure IV.6(a) Phase Portrait with Minimum Control Strategy (no-load)

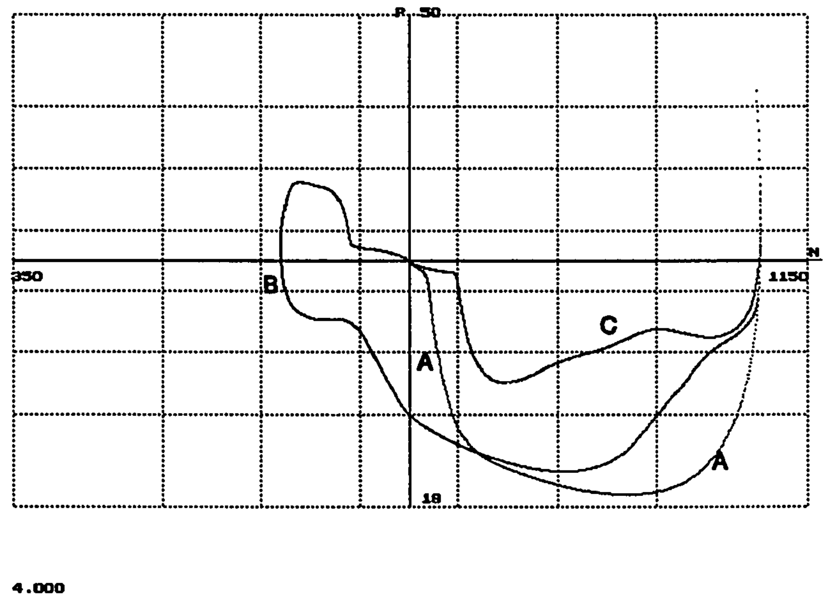


Figure IV.6(b) Phase Trajectories for various Control Strategies

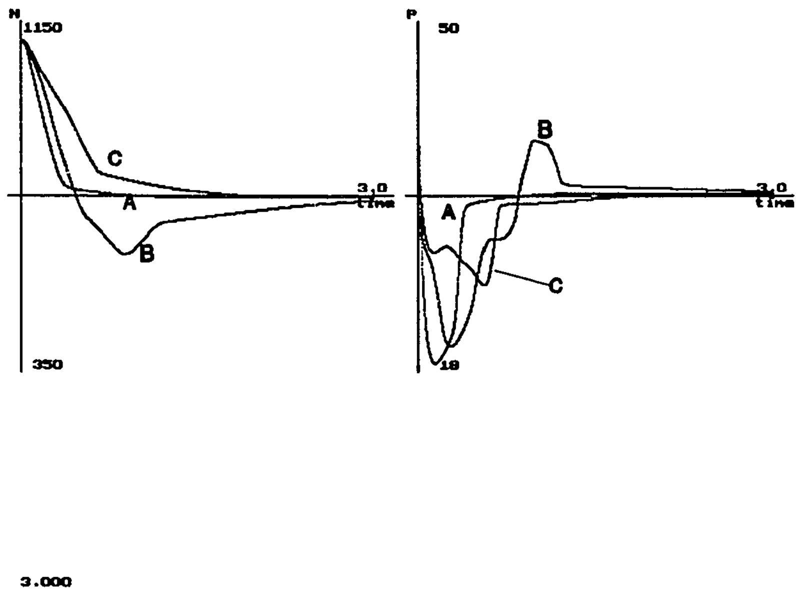


Figure IV.6(c) Time plots: Top: Shaft speed(N) and Pressure(P) Bottom: Spark Advance and Throttle Opening (A = min time B = min energy C = min squ. error)

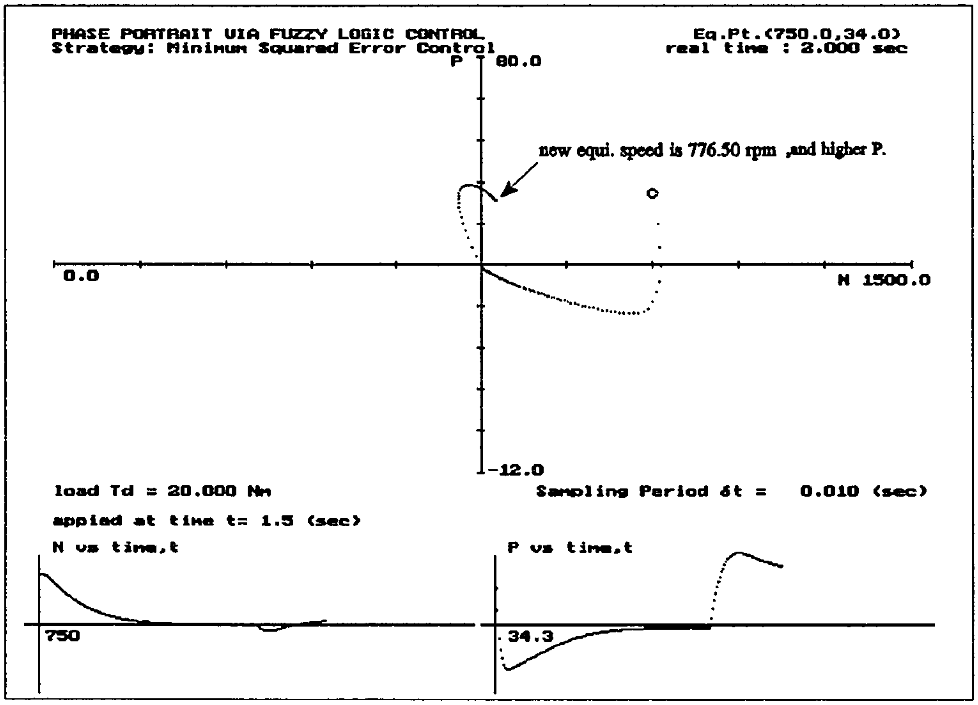


Figure IV.7 Simulation with A/C ON (min. squ. error control)

IV.6 Summary

The proposed systematic fuzzy logic control design methodology uses an approximate state space representation of a nonlinear dynamic system to develop the fuzzy rule base. The procedure guarantees completeness (since the cell-groups cover the whole state space) and optimality in some sense (since an exhaustive search technique forms the basis for choosing the final trajectories). The simulation results indicate the effectiveness of the fuzzy logic control tool in terms of stability, asymptotic convergence and robustness to external disturbances. We should note, however, that the procedure is not without some drawbacks. Complexity abounds easily as the number of inputs increases. The search can become very tasking indeed as the number of cells becomes large. Design time can also be considerably long in factoring the heuristic part of the information required for the input data. Once, though, this information is inputted, the program generates the rule base over a matter of minutes on a conventional PC platform. The tool-kit is user-friendly. Current enhancements include an analysis package aimed at assessing the performance of the closed-loop system and a hypercubic structure to accomodate systems of high dimensionality.

APPENDIX V

ELEMENTARY CONCEPTS OF LYAPUNOV STABILITY ANALYSIS

Definition V.1: *Equilibrium state*: A state x^* is an equilibrium state (or equilibrium point) of the autonomous system

$$\dot{x}(t) = f(x(t)) \tag{V.1}$$

if, once $x(t) = x^*$, it remains so for all future time. Mathematically, this says that the constant vector x^* satisfies $0 = f(x^*)$.

Lyapunov's Direct Method: The basic philosophy behind this is the mathematical extension of a fundamental physical observation: If the total energy of a system (mechanical or electrical) is continuously dissipated, then the system whether linear or nonlinear, must eventually come to a point of minimum energy and settle down there. We are led immediately here to conclude that we can study the stability of a sytem by knowning how its **scalar energy function** varies with time, **if** we can find such an energy function.

Definition V.2: *Positive definite functions*: A scalar continuous function $V(x)$ is said to be locally positive definite if $V(0) = 0$ and, in a ball B_{R0}, $x \neq 0 \Rightarrow V(x) > 0$.

If $V(0) = 0$ and the above property holds over the whole state space, then V(x) is said to be *globally positive definite*.

Similarly, a function $V(x)$ is *negative definite* if - $V(x)$ is positive definite.

Definition V.3: *Lyapunov function*: If in a ball B_{R0}, the function $V(x)$ is positive definite and continuous partial derivatives, and if its time derivative along any state trajectory of the system (V.1) is negative semi-definite, ie., $V(x) \leq 0$, then $V(x)$ is said to be a Lyapunov function for the system.

Lyapunov Theorems for Stability

Theorem V.1: (*Local Stability*): If in a ball B_{R0}, $\exists$ a scalar function $V(x)$ with continuous first partial derivatives $\ni$

1. $V(x) > 0, x \in B_{R0}$.
2. $\dot{V}(x) \leq 0, x \in B_{R0}$,

then the equilibrium point **0** is stable. If in 2. above, $\dot{V}(x) < 0$, then the stability is assymptotic.

Theorem V.2: (*Global Stability*): Let $\exists$ a scalar function $V(x)$ with continuous first partial derivatives $\ni$

1. $V(x) > 0$
2. $\dot{V}(x) < 0$
3. $V(x) \rightarrow \infty$ as $\|x\| \rightarrow \infty$,

then the equilibrium state at the origin is globally asymptotically stable.

Detail exposition and proofs may be found in [66,67].

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VITA

Shehu Sa'id FarinWata was born in Jos, Nigeria. He attended the Federal Government College, Sokoto, Nigeria, where he graduated "Division One With Distinction" - highest honor, in the West African School Certificate (WASC), 1976. He studied briefly at the Ahmadu Bello University, School of Basic Studies, before leaving for Overseas. He studied at the Oxford Air School, Kidlington, Oxford, England, and in France. While in France, he studied at the Lycée Technique de Lannion, Brittany. He was honored to be the only student selected from that center, in the 1979 Examen Special pour Entrer a l'I.U.T. - the annual special entrance exam into the universities. He started studying electrical engineering at l'I.U.T. (Institut Université de Technologie) de Lannion, a part of Université de Rennes. He left to study in the United States in December, 1979. He received the B.S. (B.Sc.) with honors (Cum Laude) and M.S. (M.Sc.) in electrical engineering from the University of Detroit, Detroit, Michigan in 1983 and 1985, respectively. He went to work as a Project Engineer with the General Motors Corporation from May 1985 to August, 1987. He worked in automotive navigation systems at the General Motors Milford Proving Ground, and later, at the General Motors Technical Center, Warren, MI. He began graduate studies at the Georgia Institute of Technology (Georgia Tech), Atlanta, Georgia in 1988, under a CIMS Fellowship. While a graduate student, he held co-op positions with GE FANUC Automation, Charlottesville, VA, and the Ford Electronics Division, Dearborn, MI, during September 1989-April 1990 and June-September 1992, respectively. He received the Ph.D. in Electrical Engineering from the Georgia Institute of Technology in September, 1993. He is a member of IEEE, and the national honor societies: Eta Kappa Nu (**HKN**) and Tau Beta Pi (**TBΠ**).

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